

2. $\{Y \in \mathfrak{B} \mid X f Y\}$ is a free star for every $X \in \mathfrak{A}$;

what is equal to the following:

1. $(I \sqcup J) f Y \Leftrightarrow I f Y \vee J f Y$ and not $0 f Y$ for every $Y \in \mathfrak{B}$, $I, J \in \mathfrak{A}$;
2. $X f (I \sqcup J) \Leftrightarrow X f I \vee X f J$ and not $X f 0$ for every $X \in \mathfrak{A}$, $I, J \in \mathfrak{B}$.

By the way, it implies that $f \mapsto [f]^*$ is a bijection from the set of functors from \mathcal{U}_0 to \mathcal{U}_1 into the set of multifunctors of the form $\mathcal{P}\mathcal{U}_0 \times \mathcal{P}\mathcal{U}_1$, for every sets \mathcal{U}_0 and \mathcal{U}_1 .

Now suppose f is a multifunctor of the form \mathfrak{P}^2 . Then:

1. $(I \sqcup J) f Y \Leftrightarrow I f Y \vee J f Y$ and not $0 f Y$ for every $Y, I, J \in \mathfrak{P}$;
2. $X f (I \sqcup J) \Leftrightarrow X f I \vee X f J$ and not $X f 0$. for every $X, I, J \in \mathfrak{P}$.

Thus multifunctors of the form \mathfrak{P}^2 are essentially equivalent to functors from \mathfrak{P} to \mathfrak{P} ([1]), formally: there exist a functor f' such that $[f']^* = f$.

$$E^*f = \{L \in \mathfrak{F}^2 \mid \text{up } L \subseteq f\} = \{(L_0; L_1) \mid L_0, L_1 \in \mathfrak{F}, \forall g_0 \in \text{up } L_0, g_1 \in \text{up } L_1 : (g_0; g_1) \in f\} = \{(L_0; L_1) \mid L_0, L_1 \in \mathfrak{F}, \forall g_0 \in \text{up } L_0, g_1 \in \text{up } L_1 : g_0 [f']^* g_1\} = \{(L_0; L_1) \mid L_0, L_1 \in \mathfrak{F}, L_0 [f'] L_1\} = [f'].$$

Thus:

1. $(I \sqcup J) (E^*f) Y \Leftrightarrow I (E^*f) Y \vee J (E^*f) Y$ and not $0 (E^*f) Y$ for every $Y, I, J \in \mathfrak{F}$;
2. $X (E^*f) (I \sqcup J) \Leftrightarrow X (E^*f) I \vee X (E^*f) J$ and not $X (E^*f) 0$ for every $X, I, J \in \mathfrak{F}$.

that is E^*f is a multifunctor of the form \mathfrak{F}^2 .

References

- [1] Victor Porton. Functors and relocks. At <http://www.mathematics21.org/binaries/functors-relocks.pdf>.
- [2] Victor Porton. Filters on posets and generalizations. *International Journal of Pure and Applied Mathematics*, 74(1):55–119, 2012.