Definition 8 ***Infinitary pre-multifuncoid*** *is such an n-ary multifuncoid that n is infinite;* ***finitary pre-multifuncoid*** *is such an n-ary multifuncoid that n is finite.*

Definition 9 *Let $\mathfrak{A}$ is a family of join-semilattices. A* ***completary multifuncoid*** *of the form $\mathfrak{A}$ is an $f \in \mathscr{P} \prod_{i \in \operatorname{dom} \mathfrak{A}} \mathfrak{A}_i$ such that*

1. *$L_0 \sqcup L_1 \in f \Leftrightarrow \exists c \in \{0,1\}^n : \left(\lambda i \in n : L_{c(i)} i\right) \in f$ for every $L_0, L_1 \in \prod \mathfrak{A}$.*
2. *If $L \in \prod \mathfrak{A}$ and $L_i = 0^{\mathfrak{A}_i}$ for some i then $\neg f L$.*

Proposition 3 *A completary multifuncoid is a multifuncoid.*

Proof Let f is a completary multifuncoid.

Let $K \in \prod_{i \in (\operatorname{dom} \mathfrak{A}) \setminus \{i\}} \mathfrak{A}_i$. Let $L_0 = K \cup \{(i; X_0)\}$, $L_1 = K \cup \{(i; X_1)\}$ for some $X_0, X_1 \in \mathfrak{A}_i$. Then $X_0 \sqcup X_1 \in (\operatorname{val} f)_i K \Leftrightarrow L_0 \sqcup L_1 \in f \Leftrightarrow \exists k \in \{0,1\} : K \cup \{(i; X_k)\} \in f \Leftrightarrow K \cup \{(i; X_0)\} \in f \vee K \cup \{(i; X_1)\} \in f \Leftrightarrow X_0 \in (\operatorname{val} f)_i K \vee X_1 \in (\operatorname{val} f)_i K$.

So $(\operatorname{val} f)_i K$ is a free star (taken in account that $K_i = 0^{\mathfrak{A}_i} \Rightarrow f \notin K$).

It remained to prove that f is an upper set. Let $L_0 \leq L_1$ for some $L_0, L_1 \in \prod \mathfrak{A}$ and $L_0 \in f$. Then taking $c = n \times \{0\}$ we get $\lambda i \in n : L_{c(i)} i = \lambda i \in n : L_0 i = L_0 \in f$ and thus $L_1 = L_0 \sqcup L_1 \in f$. □

Proposition 4 *Every finitary pre-multifuncoid is completary.*

Proof $\exists c \in \{0,1\}^n : (\lambda i \in n : L_{c(i)} i) \in f \Leftrightarrow$
$\exists c \in \{0,1\}^{n-1} : (\{(n-1; L_0(n-1))\} \cup \{(i; L_{c(i)} i) \mid i \in n-1\} \in f \vee \{(n-1; L_1(n-1))\} \cup \{(i; L_{c(i)} i) \mid i \in n-1\} \in f) \Leftrightarrow \exists c \in \{0,1\}^{n-1} : \{(n-1; L_0(n-1) \sqcup L_1(n-1))\} \cup \{(i; L_{c(i)} i) \mid i \in n-1\} \in f \Leftrightarrow \ldots \Leftrightarrow \{(i; L_0 i \sqcup L_1 i) \mid i \in n\} \in f$. □

Theorem 1 *For finite n the following are the same:*

1. *pre-multifuncoids;*
2. *multifuncoids;*
3. *completary multifuncoids.*

Proof f is a finitary pre-multifuncoid $\Rightarrow$ f is a finitary completary multifuncoid.

f is a finitary completary multifuncoid $\Rightarrow$ f is a finitary multifuncoid.

f is a finitary multifuncoid $\Rightarrow$ f is a finitary pre-multifuncoid. □

As it will be clear from below, (finitary) multifuncoids are a generalization of funcoids [1].

I will denote $\mathfrak{A}\mathsf{FCD}$ the set of multifuncoids for a finite family $\mathfrak{A}$ of join-semilattices.