1.2 Filtrators and upgrading

Definition 1 *A **filtrator** is a pair* $(\mathfrak{A}; \mathfrak{Z})$ *of a poset* $\mathfrak{A}$ *and its subset* $\mathfrak{Z}$.

See [2] for a detailed study of filtrators.

Having fixed a filtrator, we define:

Definition 2 $\operatorname{up} x = \{ Y \in \mathfrak{Z} \quad | \quad Y \geqslant x \}$ *for every* $X \in \mathfrak{A}$.

Definition 3 $E^* K = \{ L \in \mathfrak{A} \quad | \quad \operatorname{up} L \subseteq K \}$ *(**upgrading** the set* K*) for every* $K \in \mathscr{P}\mathfrak{Z}$.

1.3 Multifuncoids

Definition 4 *A **free star** on a join-semilattice* $\mathfrak{A}$ *with least element* 0 *is a set* S *such that* $0 \notin S$ *and*

$$\forall A, B \in \mathfrak{A} : (A \sqcup B \in S \Leftrightarrow A \in S \vee B \in S) .$$

I will denote the set of free stars on $\mathfrak{A}$ as $\mathfrak{A}\operatorname{Star}$.

Let n be a set. As an example, n may be an ordinal, n may be a natural number, considered as a set by the formula $n = \{0, \ldots, n-1\}$. Let $\mathfrak{A} = \mathfrak{A}_{i \in n}$ is a family of posets indexed by the set n.

Definition 5 *Let* $f \in \mathscr{P} \prod \mathfrak{A}$*,* $i \in \operatorname{dom} \mathfrak{A}$*,* $L \in \prod \mathfrak{A}|_{(\operatorname{dom} \mathfrak{A}) \setminus \{i\}}$.

$$(\operatorname{val} f)_i L = \{ X \in \mathfrak{A}_i \quad | \quad L \cup \{(i; X)\} \in f \} .$$

*("*val*" is an abbreviation of the word "value".)*

Proposition 1 f *can be restored knowing* $(\operatorname{val} f)_i$ *for some* $i \in n$.

Proof $f = \{ K \in \prod \mathfrak{A} \quad | \quad K \in f \} =$
$\left\{ L \cup \{(i; X)\} \quad | \quad L \in \prod \mathfrak{A}|_{(\operatorname{dom} \mathfrak{A}) \setminus \{i\}}, X \in \mathfrak{A}_i, L \cup \{(i; X)\} \in f \right\} =$
$\left\{ L \cup \{(i; X)\} \quad | \quad L \in \prod \mathfrak{A}|_{(\operatorname{dom} \mathfrak{A}) \setminus \{i\}}, X \in (\operatorname{val} f)_i L \right\}$. □

Definition 6 *Let* $\mathfrak{A}$ *is a family of join-semilattices. A **pre-multidimensional funcoid** (or **pre-multifuncoid** for short) of the form* $\mathfrak{A}$ *is an* $f \in \mathscr{P} \prod \mathfrak{A}$ *such that we have that:* $(\operatorname{val} f)_i L$ *is a free star for every* $i \in \operatorname{dom} \mathfrak{A}$*,* $L \in \prod \mathfrak{A}|_{(\operatorname{dom} \mathfrak{A}) \setminus \{i\}}$.

Definition 7 *A **multidimensional funcoid** (or **multifuncoid** for short) is a pre-multifuncoid which is an upper set.*

Proposition 2 *If* $L \in \prod \mathfrak{A}$ *and* $L_i = 0^{\mathfrak{A}_i}$ *for some* i *then* $L \notin f$ *if* f *is a pre-multifuncoid.*

Proof Let $K = L|_{\operatorname{dom} \mathfrak{A} \setminus \{i\}}$. We have $0 \notin (\operatorname{val} f)_i K$; $K \cup \{(i; 0)\} \notin f$; $L \notin f$. □