On Todd Trimble's notes on "topogeny"

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This short article is written as a response on Todd Trimble's notes:

http://ncatlab.org/toddtrimble/published/topogeny

In this my article I am going to reprise these Todd's theorems which are new for me, converted into terminology and notation of my book [1] and of

http://www.mathematics21.org/binaries/rewrite-plan.pdf

Now this article is a partial draft. I am going to integrate materials of this article into my book.

1 Misc

The following theorem is a strenghtening suggested by Todd of my theorem:

Theorem 1. $\bigsqcap^{\mathfrak{F}} S = \bigcup \{\uparrow(K_0 \sqcap^3 \ldots \sqcap^3 K_n) \mid K_i \in \bigcup S \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\}$ for every nonempty set S of filters on a meet-semilattice. [TODO: Strengthen my theorems requiring distributivity of lattice with this result which does not require it even to be a lattice.]

Proof. It follows from the fact that

$$\bigsqcap^{\mathfrak{F}} S = \bigsqcap^{\mathfrak{F}} \{K_0 \sqcap^3 \ldots \sqcap^3 K_n \mid K_i \in \bigcup S \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\}$$

and that $\{K_0 \sqcap^3 \ldots \sqcap^3 K_n \mid K_i \in \bigcup S \text{ where } i = 0, \ldots, n \text{ for } n \in \mathbb{N}\}$ is a filter base. □

Definition 2. A complete lattice is *co-compact* iff $\bigsqcap S = 0$ for a set S of elements of this lattice implies that there is its finite subset $T \subseteq S$ such that $\bigsqcap T = 0$. [TODO: Remove the requirement to have least.]

Proposition 3. The poset of filters on a meet-semilattice $\mathfrak{Z}$ with least element is co-compact.

Proof. If $0 \in \bigsqcap^{\mathfrak{F}} S$ then there are $K_i \in \bigcup S$ such that $0 \in \uparrow(K_0 \sqcap^3 \ldots \sqcap^3 K_n)$ that is $K_0 \sqcap^3 \ldots \sqcap^3 K_n = 0$ from which easily follows $\mathcal{F}_0 \sqcap^{\mathfrak{F}} \ldots \sqcap^{\mathfrak{F}} \mathcal{F}_n = 0$ for some $\mathcal{F}_i \in S$. □

Proposition 4. $\mathcal{X} [f] \bigsqcap S \Leftrightarrow \exists \mathcal{Y} \in S: \mathcal{X} [f] \mathcal{Y}$ if S is a generalized filter base on $\operatorname{Dst} f$. [TODO: Pointfree funcoids.]

Proof. $\mathcal{X} [f] \bigsqcap S \Leftrightarrow \bigsqcap S \sqcap \langle f \rangle \mathcal{X} \neq 0 \Leftrightarrow \bigsqcap \langle \langle f \rangle \mathcal{X} \sqcap \rangle^* S \neq 0 \Leftrightarrow$ (by properties of generalized filter bases)$\Leftrightarrow \exists \mathcal{Y} \in \langle \langle f \rangle \mathcal{X} \sqcap \rangle^* S: \mathcal{Y} \neq 0 \Leftrightarrow \exists \mathcal{Y} \in S: \langle f \rangle \mathcal{X} \sqcap \mathcal{Y} \neq 0 \Leftrightarrow \exists \mathcal{Y} \in S: \mathcal{X} [f] \mathcal{Y}$. □

The following theorem was easy to prove, but I would not discovered it without Todd's help:

Theorem 5. A function $\varphi: \mathfrak{F}(A) \rightarrow \mathfrak{F}(B)$ preserves finite joins and filtered meets iff there exists a funcoid f such that $\langle f \rangle = \varphi$. [TODO: Define filtered meets. Say about empty join 0.]