But $x \sqcup y \in a \Leftrightarrow x \in a \vee y \in a$ because a is an ultrafilter. So, the formula (1) holds, and we have proved that f is really a staroid.

Take X be the constant function with value A and Y be the constant function with value B. $\forall p \in \operatorname{GR} f: p \not\asymp X$ because $p_i \cap X_i \in a$; so $\operatorname{GR} f \subseteq \operatorname{GR} \prod^{\mathsf{Strd}} X$ that is $f \sqsubseteq \prod^{\mathsf{Strd}} X$.

Finally, $Y \sqcap X \notin \operatorname{GR} f$ because $X \sqcap Y = \lambda i \in U: A \cap B$. □

Some conjectures similar to the above example:

Conjecture 82. There exists a completary staroid f and an indexed family X of principal filters (with arity $f = \operatorname{dom} X$ and $(\operatorname{form} f)_i = \operatorname{Base}(X_i)$ for every $i \in \operatorname{arity} f$), such that $f \sqsubseteq \prod^{\mathsf{Strd}} X$ and $Y \sqcap X \notin \operatorname{GR} f$ for some $Y \in \operatorname{GR} f$.

Conjecture 83. There exists a staroid f and an indexed family x of ultrafilters (with arity $f = \operatorname{dom} x$ and $(\operatorname{form} f)_i = \operatorname{Base}(x_i)$ for every $i \in \operatorname{arity} f$), such that $f \sqsubseteq \prod^{\mathsf{Strd}} x$ and $Y \sqcap x \notin \operatorname{GR} f$ for some $Y \in \operatorname{GR} f$.

Other conjectures:

Conjecture 84. If staroid $0 \neq f \sqsubseteq a^n_{\mathsf{Strd}}$ for an ultrafilter a and an index set n, then $n \times \{a\} \in \operatorname{GR} f$. (Can it be generalized for arbitrary staroidal products?)

Conjecture 85. The following posets are atomic:

1. anchored relations on powersets;
2. staroids on powersets;
3. completary staroids on powersets.

Conjecture 86. The following posets are atomistic:

1. anchored relations on powersets;
2. staroids on powersets;
3. completary staroids on powersets.

The above conjectures seem difficult, because we know almost nothing about structure of atomic staroids.

Conjecture 87. A staroid on powersets is principal iff it is complete in every argument.

Conjecture 88. If a is an ultrafilter, then $\operatorname{id}^{\mathsf{Strd}}_{a[n]}$ is an atom of the lattice of:

1. anchored relations of the form $(\mathscr{P}\operatorname{Base}(a))^n$;
2. staroids of the form $(\mathscr{P}\operatorname{Base}(a))^n$;
3. completary staroids of the form $(\mathscr{P}\operatorname{Base}(a))^n$.

Conjecture 89. If a is an ultrafilter, then $\upuparrows \operatorname{id}^{\mathsf{Strd}}_{a[n]}$ is an atom of the lattice of:

1. anchored relations of the form $\mathfrak{F}(\operatorname{Base}(a))^n$;
2. staroids of the form $\mathfrak{F}(\operatorname{Base}(a))^n$;
3. completary staroids of the form $\mathfrak{F}(\operatorname{Base}(a))^n$.

Informal problem: Formulate and prove associativity of staroidal product.

Bibliography

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