2. $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ if $(\mathfrak{A};\mathfrak{Z})$ is a primary filtrator over a distributive lattice.

**Proof.**

1. $L \in \mathrm{GR} \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow L \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \mathrm{MEET}(\{L_i \mid i \in n\} \cup \{\mathcal{A}\}) \Leftrightarrow \bigsqcap^{\mathfrak{A}}_{i\in n} L_i \sqcap \mathcal{A} \neq 0 \Leftrightarrow$ (by finiteness)$\Leftrightarrow \bigsqcap^{\mathfrak{Z}}_{i\in n} L_i \sqcap \mathcal{A} \neq 0 \Leftrightarrow L \in \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ for every $L \in \prod \mathfrak{Z}$.

2. $L \in \mathrm{GR} \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \mathrm{up}\, L \subseteq \mathrm{GR}\,\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \forall K \in \mathrm{up}\, L \colon K \in \mathrm{GR}\,\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \forall K \in \mathrm{up}\, L \colon \bigsqcap^{\mathfrak{Z}}_{i\in n} K_i \in \partial \mathcal{A} \Leftrightarrow \forall K \in \mathrm{up}\, L \colon \bigsqcap^{\mathfrak{Z}}_{i\in n} K_i \not\asymp \mathcal{A} \Leftrightarrow$ (by finiteness and theorem 4.44??)$\Leftrightarrow \forall K \in \mathrm{up}\, L \colon \bigsqcap^{\mathfrak{A}}_{i\in n} K_i \not\asymp \mathcal{A} \Leftrightarrow \mathcal{A} \in \bigcap \langle\star\rangle \left\{ \bigsqcap^{\mathfrak{A}}_{i\in n} K_i \mid K \in \mathrm{up}\, L \right\} \Leftrightarrow$ (by the formula for finite meet of filters, theorem 4.111??)$\Leftrightarrow \mathcal{A} \in \bigcap \langle\star\rangle \bigsqcap^{\mathfrak{A}}_{i\in n} L_i \Leftrightarrow \forall K \in \bigsqcap^{\mathfrak{A}}_{i\in n} L_i \colon \mathcal{A} \in \star K \Leftrightarrow \forall K \in \bigsqcap^{\mathfrak{A}}_{i\in n} L_i \colon \mathcal{A} \not\asymp K \Leftrightarrow$ (by separability of core, theorem 4.112??)$\Leftrightarrow \bigsqcap^{\mathfrak{A}}_{i\in n} L_i \not\asymp \mathcal{A} \Leftrightarrow L \in \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$. □

**Proposition 78.** Let $(\mathfrak{A};\mathfrak{Z})$ be a finitely meet closed filtrator. $\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ and $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ are the same for finite $n$.

**Proof.** Because $\bigsqcap^{\mathfrak{Z}}_{i\in\mathrm{dom}\, L} L_i = \bigsqcap^{\mathfrak{A}}_{i\in\mathrm{dom}\, L} L_i$ for finitary $L$. □

# 8 Counter-examples and conjectures

The following example shows that the theorem 33 can't be strenghtened:

**Example 79.** For some multifuncoid $f$ on powersets complete in argument $k$ the following formula is false:

$\langle f\rangle_l (L \cup \{(k; \bigsqcup X)\}) = \bigsqcup_{x\in X} \langle f\rangle_l (L \cup \{(k;x)\})$ for every $X \in \mathscr{P}\mathfrak{P}_k$, $L \in \prod_{i\in(\mathrm{arity}\, f)\setminus\{k,l\}} \mathfrak{F}_i$.

**Proof.** Consider multifuncoid $f = \Lambda\, \mathrm{id}^{\mathsf{Strd}}_{\uparrow U[3]}$ where $U$ is an infinite set (of the form $\mathfrak{P}^3$) and $L = (Y)$ where $Y$ is a nonprincipal filter on $U$.

$\langle f\rangle_0 (L \cup \{(k; \bigsqcup X)\}) = Y \sqcap \bigsqcup X$;

$\bigsqcup_{x\in X} \langle f\rangle_0 (L \cup \{(k;x)\}) = \bigsqcup_{x\in X} (Y \sqcap x)$.

It can be $Y \sqcap \bigsqcup X = \bigsqcup_{x\in X} (Y \sqcap x)$ only if $Y$ is principal: Really: $Y \sqcap \bigsqcup X = \bigsqcup_{x\in X} (Y \sqcap x)$ implies $Y \not\asymp \bigsqcup X \Rightarrow \bigsqcup_{x\in X} (Y \sqcap x) \neq 0 \Rightarrow \exists x \in X \colon Y \not\asymp x$ and thus $Y$ is principal. But we claimed above that it is nonprincipal. □

**Example 80.** There exists a staroid $f$ and an indexed family $X$ of principal filters (with arity $f = \mathrm{dom}\, X$ and $(\mathrm{form}\, f)_i = \mathrm{Base}(X_i)$ for every $i \in \mathrm{arity}\, f$), such that $f \sqsubseteq \prod^{\mathsf{Strd}} X$ and $Y \sqcap X \notin \mathrm{GR}\, f$ for some $Y \in \mathrm{GR}\, f$.

**Remark 81.** Such examples obviously do not exist if both $f$ is a principal staroid and $X$ and $Y$ are indexed families of principal filters (because for powerset algebras staroidal product is equivalent to Cartesian product). This makes the above example inspired.

**Proof.** (Monroe Eskew) Let $a$ be any (trivial or nontrivial) ultrafilter on an infinite set $U$. Let $A, B \in a$ be such that $A \cap B \subset A, B$. In other words, $A$, $B$ are arbitrary nonempty sets such that $\emptyset \neq A \cap B \subset A, B$ and $a$ be an ultrafilter on $A \cap B$.

Let $f$ be the staroid whose graph consists of functions $p\colon U \to a$ such that either $p(n) \supseteq A$ for all but finitely many $n$ or $p(n) \supseteq B$ for all but finitely many $n$. Let's prove $f$ is really a staroid.

It's obvious $px \neq \emptyset$ for every $x \in U$. Let $k \in U$, $L \in a^{U\setminus\{k\}}$. It is enough (taking symmetry into account) to prove that

$$L \cup \{(k; x \sqcup y)\} \in \mathrm{GR}\, f \Leftrightarrow L \cup \{(k;x)\} \in \mathrm{GR}\, f \vee L \cup \{(k;y)\} \in \mathrm{GR}\, f. \qquad (1)$$

Really, $L \cup \{(k; x \sqcup y)\} \in \mathrm{GR}\, f$ iff $x \sqcup y \in a$ and $L(n) \supseteq A$ for all but finitely many $n$ or $L(n) \supseteq B$ for all but finitely many $n$; $L \cup \{(k;x)\} \in \mathrm{GR}\, f$ iff $x \in a$ and $L(n) \supseteq A$ for all but finitely many $n$ or $L(n) \supseteq B$; and similarly for $y$.