

Proposition 51. $Y \in \text{GR id}_{\mathcal{A}[n]}^{\text{Strd}} \Leftrightarrow \forall A \in \mathcal{A}: Y \in \text{GR} \uparrow^{\text{Strd}} \text{id}_{A[n]}$ for every filter \mathcal{A} on a powerset and $Y \in \mathfrak{P}^n$.

Proof. Take $Y = \uparrow^{3^n} X$.

$\forall A \in \mathcal{A}: Y \in \text{GR} \uparrow^{\text{Strd}} \text{id}_{A[n]} \Leftrightarrow \forall A \in \mathcal{A}: \uparrow^{3^n} X \in \text{GR} \uparrow^{\text{Strd}} \text{id}_{A[n]} \Leftrightarrow \forall A \in \mathcal{A}: \prod X \not\star \text{id}_{A[n]} \Leftrightarrow \uparrow^{3^n} X \in \text{GR id}_{\mathcal{A}[n]}^{\text{Strd}} \Leftrightarrow Y \in \text{GR id}_{\mathcal{A}[n]}^{\text{Strd}}$. \square

Proposition 52. $\uparrow^{3^n} X \in \text{GR id}_{a[n]}^{\text{Strd}} \Leftrightarrow \forall A \in a \exists t \in A \forall i \in n: t \in X_i$.

Proof. $\uparrow^{3^n} X \in \text{GR id}_{a[n]}^{\text{Strd}} \Leftrightarrow \exists A \in a \exists t \in A: n \times \{t\} \in \prod X \Leftrightarrow \forall A \in a \exists t \in A \forall i \in n: t \in X_i$. \square

6.5 Relationships between big and small identity staroids

Definition 53. $a_{\text{Strd}}^n = \prod_{i \in n}^{\text{Strd}} a$ for every element a of a poset and an index set n .

Proposition 54. $\uparrow \text{id}_{a[n]}^{\text{Strd}} \sqsubseteq \text{ID}_{a[n]}^{\text{Strd}} \sqsubseteq a_{\text{Strd}}^n$ for every filter a (on any distributive lattice) and an index set n .

Proof.

$\text{GR} \uparrow \text{id}_{a[n]}^{\text{Strd}} \sqsubseteq \text{GR ID}_{a[n]}^{\text{Strd}}$. $\mathcal{L} \in \text{GR} \uparrow \text{id}_{a[n]}^{\text{Strd}} \Leftrightarrow \text{up } \mathcal{L} \subseteq \text{GR id}_{a[n]}^{\text{Strd}} \Leftrightarrow \forall L \in \text{up } \mathcal{L}: L \in \text{GR id}_{a[n]}^{\text{Strd}} \Leftrightarrow$ (proposition 4.112??) $\Leftrightarrow \forall L \in \text{up } \mathcal{L} \forall A \in \text{up } a: \prod_{i \in n}^3 L_i \not\star A \Leftrightarrow \forall L \in \text{up } \mathcal{L} \forall A \in \text{up } a: \prod_{i \in n}^3 L_i \cap A \neq \emptyset \Rightarrow \bigcup_{i \in n} \mathcal{L}_i \cup a$ has finite intersection property $\Leftrightarrow \mathcal{L} \in \text{GR ID}_{a[n]}^{\text{Strd}}$.

$\text{GR ID}_{a[n]}^{\text{Strd}} \sqsubseteq \text{GR } a_{\text{Strd}}^n$. $\mathcal{L} \in \text{GR ID}_{a[n]}^{\text{Strd}} \Leftrightarrow \text{MEET}(\{\mathcal{L}_i \mid i \in n\} \cup \{a\}) \Rightarrow \forall i \in a: \mathcal{L}_i \not\star a \Leftrightarrow \mathcal{L} \in \text{GR } a_{\text{Strd}}^n$. \square

Proposition 55. $\uparrow \text{id}_{a[a]}^{\text{Strd}} \sqsubseteq \text{ID}_{a[a]}^{\text{Strd}} = a_{\text{Strd}}^a$ for every nontrivial ultrafilter a on a set.

Proof.

$\text{GR} \uparrow \text{id}_{a[a]}^{\text{Strd}} \neq \text{GR ID}_{a[a]}^{\text{Strd}}$. Let $\mathcal{L}_i = \uparrow^{\text{Base}(a)} i$. Then trivially $\mathcal{L} \in \text{GR ID}_{a[a]}^{\text{Strd}}$. But to disprove $\mathcal{L} \in \text{GR} \uparrow \text{id}_{a[a]}^{\text{Strd}}$ it's enough to show $L \notin \text{GR id}_{a[a]}^{\text{Strd}}$ for some $L \in \text{up } \mathcal{L}$. Really, take $L_i = \mathcal{L}_i = \uparrow^{\text{Base}(a)} i$. Then $L \in \text{GR id}_{a[a]}^{\text{Strd}} \Leftrightarrow \forall A \in a \exists t \in A \forall i \in a: t \in i$ what is clearly false (we can always take $i \in a$ such that $t \notin i$ for any point t).

$\text{GR ID}_{a[a]}^{\text{Strd}} = \text{GR } a_{\text{Strd}}^a$. $\mathcal{L} \in \text{GR ID}_{a[a]}^{\text{Strd}} \Leftrightarrow \forall i \in n: \mathcal{L}_i \supseteq a \Leftrightarrow \forall i \in a: \mathcal{L}_i \not\star a \Leftrightarrow \mathcal{L} \in \text{GR } a_{\text{Strd}}^a$. \square

Corollary 56. a_{Strd}^a isn't an atom when a is a nontrivial ultrafilter.

Corollary 57. Staroidal product of an infinite indexed family of ultrafilters may be non-atomic.

Proposition 58. $\text{id}_{a[n]}^{\text{Strd}}$ is determined by the value of $\uparrow \text{id}_{a[n]}^{\text{Strd}}$. Moreover $\text{id}_{a[n]}^{\text{Strd}} = \downarrow \uparrow \text{id}_{a[n]}^{\text{Strd}}$.

Proof. Use general properties of upgrading and downgrading (proposition 17.63??). \square

Lemma 59. $\mathcal{L} \in \text{GR ID}_{a[n]}^{\text{Strd}}$ iff $\bigcup_{i \in n} \mathcal{L}_i \cup a$ has finite intersection property (for primary filtrators).

Proof. $\mathcal{L} \in \text{GR ID}_{a[n]}^{\text{Strd}} \Leftrightarrow \prod_{i \in n} \mathcal{L} \cap a \neq 0^{\mathfrak{F}} \Leftrightarrow \forall X \in \prod_{i \in n} \mathcal{L} \cap a: X \neq \emptyset$ what is equivalent of $\bigcup_{i \in n} \mathcal{L}_i \cup a$ having finite intersection property. \square

Proposition 60. $\text{ID}_{a[n]}^{\text{Strd}}$ is determined by the value of $\downarrow \text{ID}_{a[n]}^{\text{Strd}}$, moreover $\text{ID}_{a[n]}^{\text{Strd}} = \uparrow \downarrow \text{ID}_{a[n]}^{\text{Strd}}$ (for primary filtrators).

Proof. $\mathcal{L} \in \uparrow \downarrow \text{ID}_{a[n]}^{\text{Strd}} \Leftrightarrow \text{up } \mathcal{L} \subseteq \downarrow \text{ID}_{a[n]}^{\text{Strd}} \Leftrightarrow \text{up } \mathcal{L} \subseteq \text{ID}_{a[n]}^{\text{Strd}} \Leftrightarrow \forall L \in \text{up } \mathcal{L}: L \in \text{ID}_{a[n]}^{\text{Strd}} \Leftrightarrow \forall L \in \text{up } \mathcal{L}: \prod_{i \in n} L_i \cap a \neq 0^{\mathfrak{F}} \Leftrightarrow \bigcup_{i \in n} \mathcal{L}_i \cup a$ has finite intersection property \Leftrightarrow (lemma) $\Leftrightarrow \mathcal{L} \in \text{GR ID}_{a[n]}^{\text{Strd}}$. \square