It is equivalent (proposition 22 and the fact that $[f]$ is an upper set) to $\langle f\rangle_k(L\cup\{(l;Y)\})$ being a principal filter and thus $(\operatorname{val}[f])_l L$ being a complete free star.

(1)⇒(3). $Y\not\asymp\langle f\rangle_l(L\cup\{(k;\bigsqcup X)\})\Leftrightarrow\bigsqcup X\not\asymp\langle f\rangle_k(L\cup\{(l;Y)\})\Leftrightarrow\exists x\in X: x\not\asymp\langle f\rangle_k(L\cup\{(l;Y)\})\Leftrightarrow\exists x\in X: Y\not\asymp\langle f\rangle_l(L\cup\{(k;x)\})\Leftrightarrow Y\not\asymp\bigsqcup_{x\in X}\langle f\rangle_l(L\cup\{(k;x)\})$ for every principal Y. □

6 Identity staroids and multifuncoids

6.1 Identity relations

Denote $\operatorname{id}_{A[n]}=\{(\lambda i\in n: x)\mid x\in A\}=\{n\times\{x\}\mid x\in A\}$ the n-ary identity relation on a set A (for each index set n).

Proposition 34. $\prod X\not\asymp\operatorname{id}_{A[n]}\Leftrightarrow\bigcap_{i\in n}X_i\cap A\neq\emptyset$.

Proof. $\prod X\not\asymp\operatorname{id}_{A[n]}\Leftrightarrow\exists t\in A: n\times\{t\}\in\prod X\Leftrightarrow\exists t\in A\forall i\in n: t\in X_i\Leftrightarrow\bigcap_{i\in n}X_i\cap A\neq\emptyset$. □

6.2 Universal definitions of identity staroids

Consider a filtrator $(\mathfrak{A};\mathfrak{Z})$ and $\mathcal{A}\in\mathfrak{A}$.

I will define below *small identity staroids* $\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ and *big identity staroids* $\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$. That they are really staroids and even completary staroids (under certain conditions) is proved below.

Definition 35. Consider a filtrator $(\mathfrak{A};\mathfrak{Z})$. Let $\mathfrak{Z}$ be a complete lattice. Let $\mathcal{A}\in\mathfrak{A}$, let n be an index set.

$\operatorname{form}\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}=\mathfrak{Z}^n$; $L\in\operatorname{GR}\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\prod^{\mathfrak{Z}}_{i\in n}L_i\in\partial\mathcal{A}$.

Obvious 36. $X\in\operatorname{GR}\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\forall A\in\operatorname{up}\mathcal{A}:\prod^{\mathfrak{Z}}_{i\in n}X_i\sqcap A\neq 0$ if our filtrator is with separable core.

Definition 37. The subset X of a poset $\mathfrak{A}$ *has a nontrivial lower bound* (I denote this predicate as $\operatorname{MEET}(X)$) iff there is nonleast $a\in\mathfrak{A}$ such that $\forall x\in X: a\sqsubseteq x$.

Definition 38. Staroid $\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ (for any $\mathcal{A}\in\mathfrak{A}$ where $\mathfrak{A}$ is a poset) is defined by the formulas:

$\operatorname{form}\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}=\mathfrak{A}^n$; $\mathcal{L}\in\operatorname{GR}\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\operatorname{MEET}(\{\mathcal{L}_i\mid i\in n\}\cup\{\mathcal{A}\})$.

Obvious 39. If $\mathfrak{A}$ is complete lattice, then $\mathcal{L}\in\operatorname{GR}\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\prod\mathcal{L}\not\asymp\mathcal{A}$.

Obvious 40. If $\mathfrak{A}$ is complete lattice and a is an atom, then $\mathcal{L}\in\operatorname{GR}\operatorname{ID}^{\mathsf{Strd}}_{a[n]}\Leftrightarrow\prod\mathcal{L}\sqsupseteq a$.

Obvious 41. If $\mathfrak{A}$ is a complete lattice then there exists a multifuncoid $\Lambda\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ such that $\left\langle\Lambda\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\right\rangle_k L=\prod_{i\in n}L_i\sqcap\mathcal{A}$ for every $k\in n$, $L\in\mathfrak{A}^{n\setminus\{k\}}$.

Proposition 42. If $(\mathfrak{A};\mathfrak{Z})$ is a meet-closed filtrator and $\mathfrak{Z}$ is a complete lattice and $\mathfrak{A}$ is a meet-semilattice. There exists a multifuncoid $\Lambda\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ such that $\left\langle\Lambda\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\right\rangle_k L=\prod^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}\mathcal{A}$ for every $k\in n$, $L\in\mathfrak{Z}^{n\setminus\{k\}}$.

Proof. We need to prove that $L\cup\{(k;X)\}\in\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\prod^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}\mathcal{A}\not\asymp^{\mathfrak{A}}X$. But

$$\prod^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}\mathcal{A}\not\asymp^{\mathfrak{A}}X\Leftrightarrow\prod^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}X\not\asymp^{\mathfrak{A}}\mathcal{A}\Leftrightarrow\prod^{\mathfrak{Z}}_{i\in n}(L\cup\{(k;X)\})_i\not\asymp^{\mathfrak{A}}\mathcal{A}\Leftrightarrow L\cup\{(k;X)\}\in\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}.$$ □

6.3 Identities are staroids

Proposition 43. Let $\mathfrak{A}$ be a complete distributive lattice and $\mathcal{A}\in\mathfrak{A}$. Then $\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a staroid.