Proof. Take

$$f = \left\{ (\mathcal{X}; \mathcal{Y}) \mid \mathcal{X} \in \mathfrak{F}(A), \mathcal{Y} \in \mathfrak{F}(B), \bigcap \mathcal{X} \text{ and } \bigcap \mathcal{Y} \text{ are infinite} \right\}$$

and

$$g = f \cup \{ (\mathcal{X}; \mathcal{Y}) \mid \mathcal{X} \in \mathfrak{F}(A), \mathcal{Y} \in \mathfrak{F}(B), \mathcal{X} \sqsupseteq a, \mathcal{Y} \sqsupseteq b \}$$

where a and b are nontrivial ultrafilters on A and B correspondingly, c is the funcoid defined by the relation

$$[c]^* = \delta = \{ (X; Y) \mid X \in \mathscr{P}A, Y \in \mathscr{P}B, X \text{ and } Y \text{ are infinite} \}.$$

First prove that f is a pseudofuncoid. The formulas $\neg(I\, f\, 0)$ and $\neg(0\, f\, I)$ are obvious. We have $\mathcal{I} \sqcup \mathcal{J}\, f\, \mathcal{K} \Leftrightarrow \bigcap\, (\mathcal{I} \sqcup \mathcal{J})$ and $\bigcap\, \mathcal{Y}$ are infinite$\Leftrightarrow \bigcap\, \mathcal{I} \cup \bigcap\, \mathcal{J}$ and $\bigcap\, \mathcal{Y}$ are infinite$\Leftrightarrow(\bigcap\, \mathcal{I}$ or $\bigcap\, \mathcal{J}$ is infinite) $\wedge \bigcap\, \mathcal{Y}$ is infinite$\Leftrightarrow(\bigcap\, \mathcal{I}$ and $\bigcap\, \mathcal{Y}$ are infinite) $\vee$ ($\bigcap\, \mathcal{J}$ and $\bigcap\, \mathcal{Y}$ are infinite) $\Leftrightarrow \mathcal{I}\, f\, \mathcal{K} \vee \mathcal{J}\, f\, \mathcal{K}$. Similarly $\mathcal{K}\, f\, \mathcal{I} \sqcup \mathcal{J} \Leftrightarrow \mathcal{K}\, f\, \mathcal{I} \vee \mathcal{K}\, f\, \mathcal{J}$. So f is a pseudofuncoid.

Let now prove that g is a pseudofuncoid. The formulas $\neg(I\, g\, 0)$ and $\neg(0\, g\, I)$ are obvious. Let $\mathcal{I} \sqcup \mathcal{J}\, g\, \mathcal{K}$. Then either $\mathcal{I} \sqcup \mathcal{J}\, f\, \mathcal{K}$ and then $\mathcal{I} \sqcup \mathcal{J}\, g\, \mathcal{K}$ or $\mathcal{I} \sqcup \mathcal{J} \sqsupseteq a$ and then $\mathcal{I} \sqsupseteq a \vee \mathcal{J} \sqsupseteq a$ thus having $\mathcal{I}\, g\, \mathcal{K} \vee \mathcal{J}\, g\, \mathcal{K}$. So $\mathcal{I} \sqcup \mathcal{J}\, g\, \mathcal{K} \Rightarrow \mathcal{I}\, g\, \mathcal{K} \vee \mathcal{J}\, g\, \mathcal{K}$. The reverse implication is obvious. We have $\mathcal{I} \sqcup \mathcal{J}\, g\, \mathcal{K} \Leftrightarrow \mathcal{I}\, g\, \mathcal{K} \vee \mathcal{J}\, g\, \mathcal{K}$ and similarly $\mathcal{K}\, g\, \mathcal{I} \sqcup \mathcal{J} \Leftrightarrow \mathcal{K}\, g\, \mathcal{I} \vee \mathcal{K}\, g\, \mathcal{J}$. So g is a pseudofuncoid.

Obviously $f \neq g$ ($a\, g\, b$ but not $a\, f\, b$).

It remains to prove $f \cap (\mathfrak{P} \times \mathfrak{P}) = g \cap (\mathfrak{P} \times \mathfrak{P}) = [c] \cap (\mathfrak{P} \times \mathfrak{P})$. Really, $f \cap (\mathfrak{P} \times \mathfrak{P}) = [c] \cap (\mathfrak{P} \times \mathfrak{P})$ is obvious. If $(\uparrow^A X; \uparrow^B Y) \in g \cap (\mathfrak{P} \times \mathfrak{P})$ then either $(\uparrow^A X; \uparrow^B Y) \in f \cap (\mathfrak{P} \times \mathfrak{P})$ or $X \in \operatorname{up} a$, $Y \in \operatorname{up} b$, so X and Y are infinite and thus $(\uparrow^A X; \uparrow^B Y) \in f \cap (\mathfrak{P} \times \mathfrak{P})$. So $g \cap (\mathfrak{P} \times \mathfrak{P}) = f \cap (\mathfrak{P} \times \mathfrak{P})$. □

Remark 8. The above counter-example shows that pseudofuncoids (and more generally, any staroids on filters) are "second class" objects, they are not full-fledged because they don't bijectively correspond to funcoids and the elegant funcoids theory does not apply to them.

From the above it follows that staroids on filters do not correspond (by restriction) to staroids on principal filters (or staroids on sets).

4 Complete staroids and multifuncoids

4.1 Complete free stars

Definition 9. Let $\mathfrak{A}$ be a poset. *Complete free stars* on $\mathfrak{A}$ are such $S \in \mathscr{P}\mathfrak{A}$ that the least element (if it exists) is not in S and for every $T \in \mathscr{P}\mathfrak{A}$

$$\forall Z \in \mathfrak{A}: (\forall X \in T: Z \sqsupseteq X \Rightarrow Z \in S) \Leftrightarrow T \cap S \neq \emptyset.$$

Obvious 10. Every complete free star is a free star.

Proposition 11. $S \in \mathscr{P}\mathfrak{A}$ where $\mathfrak{A}$ is a poset is a complete free star iff all the following:

1. The least element (if it exists) is not in S.
2. $\forall Z \in \mathfrak{A}: (\forall X \in T: Z \sqsupseteq X \Rightarrow Z \in S) \Rightarrow T \cap S \neq \emptyset$.
3. S is an upper set.

Proof.

⇒. (1) and (2) are obvious. S is an upper set because S is a free star.

⇐. We need to prove that

$$\forall Z \in \mathfrak{A}: (\forall X \in T: Z \sqsupseteq X \Rightarrow Z \in S) \Leftarrow T \cap S \neq \emptyset.$$

Let $X' \in T \cap S$. Then $\forall X \in T: Z \sqsupseteq X \Rightarrow Z \sqsupseteq X' \Rightarrow Z \in S$ because S is an upper set. □

Proposition 12. Let S be a complete lattice. $S \in \mathscr{P}\mathfrak{A}$ is a complete free star iff all the following:

1. The least element (if it exists) is not in S.