Identity Staroids

by Victor Porton

Email: porton@narod.ru
Web: http://www.mathematics21.org

1 Draft status

This text is a draft.

Read my book [1] before reading this. Well, most probably I will integrate materials from this article into my book.

2 Additional propositions

Proposition 1. $\left\{ \langle f \rangle_k X \mid X \in \operatorname{up}^{\left(\prod_{i \in n \setminus \{k\}} \mathfrak{A}_i ; \prod_{i \in n \setminus \{k\}} \mathfrak{Z}_i \right)} \mathcal{X} \right\}$ is a filter base on $\mathfrak{A}_k$ for every family $(\mathfrak{A}_i; \mathfrak{Z}_i)$ of filtrators where $i \in n$ for some index set n (provided that f is a multifuncoid of the form $\mathfrak{A}$ and $k \in n$ and every $\mathfrak{Z}_i$ for $i \in n \setminus \{k\}$ is a filter base and $\mathcal{X} \in \prod_{i \in n \setminus \{k\}} \mathfrak{A}_i$).

Proof. Let $\mathcal{K}, \mathcal{L} \in \{ \langle f \rangle_k X \mid X \in \operatorname{up} \mathcal{X} \}$. Then there exist $X, Y \in \operatorname{up} \mathcal{X}$ such that $\mathcal{K} = \langle f \rangle_k X$, $\mathcal{L} = \langle f \rangle_k Y$. We can take $Z \in \operatorname{up} \mathcal{X}$ such that $Z \sqsubseteq X, Y$. Then evidently $\langle f \rangle_k Z \sqsubseteq \mathcal{K}$ and $\langle f \rangle_k Z \sqsubseteq \mathcal{L}$ and $\langle f \rangle_k Z \in \{ \langle f \rangle_k X \mid X \in \operatorname{up} \mathcal{X} \}$. □

Proposition 2. $\langle \upuparrows f \rangle_k \mathcal{X} = \bigsqcap_{X \in \operatorname{up} \mathcal{X}} \langle f \rangle_k X$ for a filtrator $\left(\prod_{i \in n \setminus \{k\}} \mathfrak{F}_i ; \prod_{i \in n \setminus \{k\}} \mathfrak{P}_i \right)$ ($i \in n$ for some index set n) where every $\mathfrak{Z}_i$ is a boolean lattice, $k \in n$, and $\mathcal{X} \in \prod_{i \in n \setminus \{k\}} \mathfrak{F}_i$.

Proof. $\mathfrak{F}_k$ is separable by obvious 4.136??. $(\mathfrak{F}_k; \mathfrak{P}_k)$ is with separable core by theorem 4.112??.

$\mathcal{Y} \not\asymp \langle \upuparrows f \rangle_i \mathcal{X} \Leftrightarrow \mathcal{X} \cup \{ (i; \mathcal{Y}) \} \in \operatorname{GR} [\upuparrows f] \Leftrightarrow \mathcal{X} \cup \{ (i; \mathcal{Y}) \} \in \upuparrows \operatorname{GR} [f] \Leftrightarrow \operatorname{up} (\mathcal{X} \cup \{ (i; \mathcal{Y}) \}) \subseteq \operatorname{GR} [f] \Leftrightarrow \forall X \in \operatorname{up} \mathcal{X}, Y \in \operatorname{up} \mathcal{Y} \colon X \cup \{ (i; Y) \} \in \operatorname{GR} [f] \Leftrightarrow \forall X \in \operatorname{up} \mathcal{X}, Y \in \operatorname{up} \mathcal{Y} \colon Y \not\asymp \langle f \rangle_i X \Leftrightarrow \forall X \in \operatorname{up} \mathcal{X}, Y \in \operatorname{up} \mathcal{Y} \colon Y \sqcap \langle f \rangle_i X \neq 0 \Leftrightarrow \forall Y \in \operatorname{up} \mathcal{Y} \colon 0 \notin \{ Y \sqcap \langle f \rangle_i X \mid X \in \operatorname{up} \mathcal{X} \} \Leftrightarrow \forall Y \in \operatorname{up} \mathcal{Y} \colon 0 \notin \langle Y \sqcap \rangle \{ \langle f \rangle_i X \mid X \in \operatorname{up} \mathcal{X} \} \Leftrightarrow$ (by properties of generalized filter bases)$\Leftrightarrow \forall Y \in \operatorname{up} \mathcal{Y} \colon \bigsqcap \langle Y \sqcap \rangle \{ \langle f \rangle_i X \mid X \in \operatorname{up} \mathcal{X} \} \neq 0 \Leftrightarrow \forall Y \in \operatorname{up} \mathcal{Y} \colon Y \sqcap \bigsqcap \{ \langle f \rangle_i X \mid X \in \operatorname{up} \mathcal{X} \} \neq 0 \Leftrightarrow \forall Y \in \operatorname{up} \mathcal{Y} \colon Y \not\asymp \bigsqcap_{X \in \operatorname{up} \mathcal{X}} \langle f \rangle_i X \Leftrightarrow \mathcal{Y} \not\asymp \bigsqcap_{X \in \operatorname{up} \mathcal{X}} \langle f \rangle_i X$; so $\langle \upuparrows f \rangle_i \mathcal{X} = \bigsqcap_{X \in \operatorname{up} \mathcal{X}} \langle f \rangle_i X$. □

3 On pseudofuncoids

Definition 3. *Pseudofuncoid* from a set A to a set B is a relation f between filters on A and B such that:

$$\neg (I \, f \, 0), \quad \mathcal{I} \sqcup \mathcal{J} \, f \, \mathcal{K} \Leftrightarrow \mathcal{I} \, f \, \mathcal{K} \vee \mathcal{J} \, f \, \mathcal{K} \quad \text{(for every } \mathcal{I}, \mathcal{J} \in \mathfrak{F}(A), \mathcal{K} \in \mathfrak{F}(B)\text{)},$$
$$\neg (0 \, f \, I), \quad \mathcal{K} \, f \, \mathcal{I} \sqcup \mathcal{J} \Leftrightarrow \mathcal{K} \, f \, \mathcal{I} \vee \mathcal{K} \, f \, \mathcal{J} \quad \text{(for every } \mathcal{I}, \mathcal{J} \in \mathfrak{F}(B), \mathcal{K} \in \mathfrak{F}(A)\text{)}.$$

Obvious 4. Pseudofuncoid is just a staroid of the form $(\mathfrak{F}(A); \mathfrak{F}(B))$.

Obvious 5. $[f]$ is a pseudofuncoid for every funcoid f.

Example 6. If A and B are infinite sets, then there exist two different pseudofuncoids f and g from A to B such that $f \cap (\mathfrak{P} \times \mathfrak{P}) = g \cap (\mathfrak{P} \times \mathfrak{P}) = [c] \cap (\mathfrak{P} \times \mathfrak{P})$ for some funcoid c.

Remark 7. Considering a pseudofuncoid f as a staroid, we get $f \cap (\mathfrak{P} \times \mathfrak{P}) = \downdownarrows f$.