9.1 Last axiom of proximity

As the following propositions show (merge them into one theorem), the last axiom of proximity is equivalent to transitivity of funcoids:

Proposition 34. If f is a transitive, symmetric funcoid, then the last axiom of proximity holds.

Proof. $\neg(A\,[f]\,B) \Leftrightarrow \neg(A\,[f^{-1}\circ f]\,B) \Leftrightarrow \langle f\rangle B \asymp \langle f\rangle A \Leftrightarrow \exists M \in \operatorname{Ob} f: M \asymp \langle f\rangle A \wedge \overline{M} \asymp \langle f\rangle B$. □

Proposition 35. For a reflexive funcoid, the last axiom of proximity implies that it is transitive and symmetric.

Proof. Let $\neg(A\ [f]\ B)$ implies $\exists M$: $M \asymp \langle f\rangle A \ \wedge\ \overline{M} \asymp \langle f\rangle B$. Then $\neg(A\ [f]\ B)$ implies $\neg(A\ [f^{-1}\circ f]\ B)$ that is $f \sqsupseteq f^{-1}\circ f$ and thus $f = f^{-1}\circ f$. By theorem ??(about transitive endomorphisms) f is transitive and symmetric. □

So proximity spaces are the same as reflexive, symmetric, transitive funcoids.

Remove all other definitions of uniform spaces, to be defined exactly once.

10 Misc

Say that (FCD) and $\uparrow^{\mathsf{FCD}}$, $\uparrow^{\mathsf{RLD}}$ are functors.

$\bigsqcup\{F(x)\mid x\in A\} \to \bigsqcup_{x\in A} F(x)$ (first define this notation).

Define $C(A;B) = \operatorname{Mor}_C(A;B)$.

Change superflusous notation: $\uparrow^{\mathsf{FCD}(A;B)} f \to \uparrow^{\mathsf{FCD}}(A;\ B;\ f)$ and likewise for RLD. The old notation is sometimes useful as in the definition $\Delta = \bigsqcap\left\{\uparrow^{\mathfrak{F}(\mathbb{R})}(-\varepsilon;\varepsilon)\mid \varepsilon\in\mathbb{R}, \varepsilon>0\right\}$.

Proofs that $\uparrow^{\mathsf{FCD}}(g\circ f) = \uparrow^{\mathsf{FCD}} g \circ \uparrow^{\mathsf{FCD}} f$ and $\uparrow^{\mathsf{RLD}}(g\circ f) = \uparrow^{\mathsf{RLD}} g\circ\uparrow^{\mathsf{RLD}} f$.

Probably, $\operatorname{id}^{C(A)} \to 1^C_A$ and leave $\operatorname{id}^{\mathsf{FCD}}_A$ for restricted identity funcoids and reloids.

Theorem 36. A complete lattice is atomistic iff it is atomically separable.

Proof.

- $\Rightarrow$. Let our poset is atomistic. Then obviously $\operatorname{atoms} a \neq \operatorname{atoms} b$ for elements $a\neq b$.
- $\Leftarrow$. Let "atoms" be injective. Consider an element a of our poset. Let $b = \bigsqcup \operatorname{atoms} a$. Obviously $b\sqsubseteq a$ and thus $\operatorname{atoms} b \subseteq \operatorname{atoms} a$. But if $x\in\operatorname{atoms} a$ then $x\sqsubseteq b$ and thus $x\in\operatorname{atoms} b$. So $\operatorname{atoms} a = \operatorname{atoms} b$. By injectivity $a=b$ that is $a = \bigsqcup \operatorname{atoms} a$. □

Definition 37. $\bigsqcup^C X \stackrel{\text{def}}{=} \bigsqcup^{C(\operatorname{Src} X;\operatorname{Dst} X)} X$ for a morphism X of a directed multigraph C each Mor-set of which is a poset. Similarly for $\bigsqcap$, $\sqcup$, $\sqcap$.

Say: Whilst I have (mostly) throughly studied basic properties of funcoids, *staroids* (defined below) are yet much a mystery. For example, we do not know whether the set of staroids on powersets is atomic.

star-comparison.tm

cross-composition-funcoids.tm

todd-notes.tm

11 Errors

"Theorem 17.150. Anchored relations with objects being atomic posets and above defined compositions form a quasi-invertible category with star-morphisms."

It is wrong, because composition of a star-morphism m with identify morphisms may be not equal to m. In the definition of general cross-composition product we can replace quasi-invertible category with quasi-invertible pre-category.

Bibliography

[1] Victor Porton. Filters on posets and generalizations. *International Journal of Pure and Applied Mathematics*, 74(1):55–119, 2012. http://www.mathematics21.org/binaries/filters.pdf.