

**Conjecture 31.** For an injective funcoid  $f$ :

1.  $\langle f \rangle \sqcap^{\mathfrak{F}} S = \prod_{X \in S} \langle f \rangle X$  and  $S \in \mathcal{P} \mathcal{P} \text{Src } f$ .
2.  $\langle f \rangle \sqcap^{\mathfrak{F}} S = \prod_{X \in S} \langle f \rangle X$  and  $S \in \mathcal{P} \mathfrak{F} \text{Src } f$ .

Equality  $(\bigcap G) \circ f = \bigcap_{g \in G} (g \circ f)$  for every  $G$  implies that  $f$  is a function. Generalize for funcoids and reloids.

I do some research in:

- backward.pdf
- multireloids-relationships.pdf

Question: Can we restore the set of binary relations, knowing only order of  $\text{FCD}(A; B)$ ? Note that it is not the center of the lattice, as not all funcoids are in the center. Yes, it can be characterized as joins of complemented funcoids or joins of complemented atomic funcoids. Is every complemented funcoid principal? This way principality can be generalized for pointfree funcoids. The set of principal p.f. funcoids is join-closed. When filtrator of pointfree funcoids is filtered?

Should we extend filtrators with finite join/meet closed core to nullary closed (having bottom/top)? The old concept shall be named **binary join/meet closed filtrators**. These are related with up/down aligned filtrators.

$a ? b = \bigsqcup \{z \in \mathfrak{A} \mid a \sqsupseteq b \sqcap z\} = \overline{b \setminus a} = \bar{b} \sqcup a$  for filters. Also dual of second quasidifference.

Define Fréchet element for a filtrator by the formula  $\Omega = \max \{\mathcal{X} \in \mathfrak{F} \mid \text{Cor } \mathcal{X} = 0^3\}$ . (It uses the formula  $\text{Cor } \bigsqcup^{\mathfrak{F}} S = \bigsqcup^{\mathfrak{F}} \langle \text{Cor} \rangle S$  which in turn uses properties of Fréchet filter, so this would probably a circular proof.)

What about pseudocomplement filter of infinite joins and meets of filters?

Write an explicit formula for composition with a complete reloid (with the function  $F(\alpha)$  to which the complete reloid bijectively corresponds). Using this formula prove that complete reloids are meta-complete. Also for funcoids.

**Conjecture 32.**  $\delta = [f]$  for a funcoid  $f$  iff all of the following: [TODO: generalize for staroids]

1.  $\neg(0 \delta \mathcal{Y})$
2.  $\neg(\mathcal{X} \delta 0)$
3.  $(\mathcal{I} \sqcup \mathcal{J}) \delta \mathcal{K} \Leftrightarrow \mathcal{I} \delta \mathcal{K} \vee \mathcal{J} \delta \mathcal{K}$ ;
4.  $\mathcal{K} \delta (\mathcal{I} \sqcup \mathcal{J}) \Leftrightarrow \mathcal{K} \delta \mathcal{I} \vee \mathcal{K} \delta \mathcal{J}$ ;
5.  $\mathcal{X} [f] \sqcap S \Leftrightarrow \forall \mathcal{Y} \in S: \mathcal{X} [f] \mathcal{Y}$  for filtered set  $S$  of filters;
6.  $\sqcap S [f] \mathcal{Y} \Leftrightarrow \forall \mathcal{X} \in S: \mathcal{X} [f] \mathcal{Y}$  for filtered set  $S$  of filters.

**Conjecture 33.** Every funcoid is a composition of a co-complete funcoid and complete funcoid (or vice versa?) [TODO: Try to prove it using the fact that a funcoid is a join of products of ultrafilters. What's about reloids?] [TODO: If this conjecture is false, what about representing every funcoid as composition of three funcoids: complete, principal, and co-complete?]

The following are equivalent for a funcoid  $f$  (call it *strictly monovalued funcoid*):

1.  $f$  is a function restricted to a filter;
2.  $f$  corresponds to a monovalued reloid;
3. Is it equivalent to monovaluedness of  $(\text{RLD})_{\text{in}} f$  or  $(\text{RLD})_{\text{out}} f$ ? ( $(\text{RLD})_{\text{in}} f$  is not monovalued if  $f$  is an identity)
4. other? (no ideas: open question)

Use the above result for ordering of filters.

Homeomorphisms between funcoids. They are also isomorphisms between filters?

Check current.tm and other files.

Preservation of properties (reflexivity, symmetry, etc.) of funcoids and reloids by lattice operations.