We can limit to the case when L is a reloidal product. Then

$$L \in \bigcap \{\mathrm{up}(\{x\} \times^{\mathsf{RLD}} \langle g\rangle\{x\}) \mid x \in \mathrm{Src}\, f\} = \bigcap \{\{\{x\} \times Y \mid Y \in \mathrm{up}\, \langle g\rangle\{x\}\} \mid x \in \mathrm{Src}\, f\}.$$

It's enough to prove that $L \in \mathrm{up}\, g$. Really, $\forall x \in \mathrm{Src}\, f \colon \langle L\rangle^{*}\{x\} \in \mathrm{up}\, \langle g\rangle\{x\}$ because $\langle L\rangle^{*}\{x\} \supseteq \langle T\rangle^{*}\{x\}$ for

$$T = \bigcap \{\{\{x\} \times Y \mid Y \in \mathrm{up}\, G(x)\} \mid x \in \mathrm{Src}\, f\}.$$

and thus

$$\begin{aligned} \langle L\rangle^{*}\{x\} &\supseteq \\ \bigcap \{\{\langle \{x\} \times Y\rangle^{*}\{x'\} \mid x' = x, Y \in \mathrm{up}\, G(x)\} \mid x \in \mathrm{Src}\, f\} &= \\ \{Y \mid Y \in \mathrm{up}\, G(x')\} &= \\ \mathrm{up}\, G(x'). \end{aligned}$$

So $\langle L\rangle^{*}\{x\} \in \mathrm{up}\, \langle g\rangle\{x\}$ and thus $L \in \mathrm{up}\, g$. □

Corollary 27. $f \neq g \Rightarrow (\mathsf{RLD})_{\mathrm{out}} f \neq (\mathsf{RLD})_{\mathrm{out}}\, g$ for complete funcoids f and g.

Theorem 28. Composition of complete reloids is complete.

Proof. Let f, g be complete reloids. Then $(\mathsf{FCD})(g \circ f) = (\mathsf{FCD})g \circ (\mathsf{FCD})f$. Thus (because $(\mathsf{FCD})(g \circ f)$ is a complete funcoid) we have $g \circ f = (\mathsf{RLD})_{\mathrm{out}}((\mathsf{FCD})g \circ (\mathsf{FCD})f)$, but $(\mathsf{FCD})g \circ (\mathsf{FCD})f$ is a complete funcoid, thus $g \circ f$ is a complete reloid. □

Theorem 29.

1. $(\mathsf{RLD})_{\mathrm{out}}\, g \circ (\mathsf{RLD})_{\mathrm{out}} f = (\mathsf{RLD})_{\mathrm{out}}(g \circ f)$ for composable complete funcoids f and g.
2. $(\mathsf{RLD})_{\mathrm{out}}\, g \circ (\mathsf{RLD})_{\mathrm{out}} f = (\mathsf{RLD})_{\mathrm{out}}(g \circ f)$ for composable co-complete funcoids f and g.

Proof. Let f, g are composable complete funcoids.

$(\mathsf{FCD})((\mathsf{RLD})_{\mathrm{out}}\, g \circ (\mathsf{RLD})_{\mathrm{out}} f) = (\mathsf{FCD})(\mathsf{RLD})_{\mathrm{out}}\, g \circ (\mathsf{FCD})(\mathsf{RLD})_{\mathrm{out}} f = g \circ f$.

Thus (taking into account that $(\mathsf{RLD})_{\mathrm{out}}\, g \circ (\mathsf{RLD})_{\mathrm{out}} f$ is complete) we have $(\mathsf{RLD})_{\mathrm{out}}\, g \circ (\mathsf{RLD})_{\mathrm{out}} f = (\mathsf{RLD})_{\mathrm{out}}(g \circ f)$.

For co-complete funcoids it's dual. □

9 Not yet written

Continuity in metric spaces is continuity in topology spaces; uniform continuity in metric spaces is continuity in proximity and uniform spaces.

Pointfree analog of the lattice Γ. Also consider the lattice of finite unions of funcoidal products of fitlers (and generalizations).

Introduce core of a lattice $\mathsf{FCD}(\mathscr{F}(\mathfrak{A}); \mathscr{F}(\mathfrak{B}))$ as $\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$. Generalize it for staroids. Also filter on $\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ can be considered as pointfree reloids.

Uniform spaces (or proximities?) are equivalent to sets of filters? (Do tornings bijectively correspond to uniform spaces?)

Is Cor a functor for a. funcoids; b. reloids? Isn't it adjoint of $\uparrow^{\mathsf{FCD}}$ or $\uparrow^{\mathsf{RLD}}$?

Adjunction of prefunctors: http://www.sciencedirect.com/science/article/pii/0304397585900623 (free download, also Google for "pre-adjunction", also "semi" instead of "pre") Are (FCD) and $(\mathsf{RLD})_{\mathrm{in}}$ adjunct?

Definition 30. A morphism of a category each Mor-sets of which is a meet-semilattice, is *weakly metamonovalued* iff $(g \sqcap h) \circ f = (g \circ f) \sqcap (h \circ f)$. Similarly define *weakly metainjective*.

Prove that monovalued, metamonovalued, and weakly metamonovalued are the same for Rel, FCD, and RLD.

What are pointfree funcoids between $\mathscr{P}A$ and $\mathscr{P}B$?