# 8 Other new theorems

funcoids-are-filters.tm

funcoids-are-frame.tm

**Theorem 19.** The set of funcoids is with separable core.

**Proof.** Because filters on distributive lattices are with separable core. □

**Theorem 20.** The set of funcoids is with co-separable core. [TODO: For pointfree funcoids?]

**Proof.** Let $f, g \in \mathsf{FCD}(A; B)$ and $f \sqcup g = 1$. Then for every $X \in \mathscr{P}A$ we have

$$\langle f \rangle^* X \sqcup \langle g \rangle^* X = 1 \Leftrightarrow \operatorname{Cor} \langle f \rangle^* X \sqcup \operatorname{Cor} \langle g \rangle^* X = 1 \Leftrightarrow \langle \operatorname{CoCompl} f \rangle^* X \sqcup \langle \operatorname{CoCompl} g \rangle^* X = 1.$$

Thus $\langle \operatorname{CoCompl} f \sqcup \operatorname{CoCompl} g \rangle^* X = 1$;

$$f \sqcup g = 1 \Rightarrow \operatorname{CoCompl} f \sqcup \operatorname{CoCompl} g = 1. \tag{1}$$

Applying the dual of the formulas (1) to the formula (1) we get:

$$f \sqcup g = 1 \Rightarrow \operatorname{Compl} \operatorname{CoCompl} f \sqcup \operatorname{Compl} \operatorname{CoCompl} g = 1$$

that is $f \sqcup g = 1 \Rightarrow \operatorname{Cor} f \sqcup \operatorname{Cor} g = 1$. So $\mathsf{FCD}(A; B)$ is with co-separable core. [TODO: Say that the filtrator of complete funcoids is also with co-separable core.] □

**Proposition 21.** $\operatorname{Compl}\mathsf{FCD}(A; B)$ and $\operatorname{Compl}\mathsf{RLD}(A; B)$ are co-brouwerian lattices.

Draw a triangural diagram of correspondence of $\operatorname{Compl}\mathsf{FCD}(A; B)$ and $\operatorname{Compl}\mathsf{RLD}(A; B)$ and indexed families of filters.

**Proposition 22.** Every semifiltered filtrator is filtered. [TODO: The reverse implication is already proved.] [TODO: See also http://math.stackexchange.com/questions/1198368/a-question-on-order-theory-an-ordered-set-and-its-subset]

**Proof.** $a = \bigsqcap^{\mathfrak{A}} \operatorname{up} a$ is equivalent to $a$ is a greatest lower bound of $\operatorname{up} a$. That is the implication that $b$ is lower bound of $\operatorname{up} a$ implies $a \sqsupseteq b$.

$b$ is lower bound of $\operatorname{up} a$ implies $\operatorname{up} b \supseteq \operatorname{up} a$. So as it is semifiltered $a \sqsupseteq b$. □

## 8.1 Hyperfuncoids

Let $\mathfrak{A}$ is an indexed family of sets.

*Products* are $\prod A$ for $A \in \prod \mathfrak{A}$.

*Hyperfuncoids* are filters $\mathfrak{F}\Gamma$ on the lattice $\Gamma$ of all finite unions of products.

**Problem 23.** Is $\bigsqcap^{\mathsf{FCD}}$ a bijection from hyperfuncoids $\mathfrak{F}\Gamma$ to:

1. prestaroids on $\mathfrak{A}$;
2. staroids on $\mathfrak{A}$;
3. completary staroids on $\mathfrak{A}$?

If yes, is $\operatorname{up}^{\Gamma}$ defining the inverse bijection?

If not, characterize the image of the function $\bigsqcap^{\mathsf{FCD}}$ defined on $\mathfrak{F}\Gamma$.

## 8.2 Relationships between funcoids and reloids

**Lemma 24.** If $a$, $b$, $c$ are filters on powersets and $b \neq 0$, then

$$\bigsqcup^{\mathsf{RLD}} \{G \circ F \mid F \in \operatorname{atoms}^{\mathsf{RLD}}(a \times^{\mathsf{RLD}} b), G \in \operatorname{atoms}^{\mathsf{RLD}}(b \times^{\mathsf{RLD}} c)\} = a \times^{\mathsf{RLD}} c.$$