Proposition 27. $\tau(x)\,[\nu']\,\tau(y) \Leftrightarrow x\,[\nu]\,y$.

Proof. ?? □

Metasingular numbers

Let $y \in \mathrm{SLA}(\mathrm{Ob}\,\mu)$. I will denote $r(y)$ such $x \in \mathrm{Ob}\,\nu$ that $\tau(x) = y$, if such x exists.

I will call *base singular numbers* the set $\mathrm{BSN} = \mathrm{Ob}\,\nu \cup \mathrm{SLA}(\mathrm{Ob}\,\nu) \cup \mathrm{SLA}(\mathrm{SLA}(\mathrm{Ob}\,\nu)) \cup \ldots$.

[TODO: Set that this union is disjoint.]

I will call *meta-singular numbers* the set $\mathrm{MSN} = \{y \in \mathrm{SLA}(\mathrm{Ob}\,\nu) \mid \nexists x \in \mathrm{BSN}\colon y = \tau(x)\}$.

Definition 28. I call *reduced* BSN its corresponding MSN (that is r applied to our BSN a natural number of times while possible).

Definition 29. I call *reduced limit* the reduced generalized limit.

Functions with meta-singular numbers as arguments

Let f is an n-ary (n is an arbitrary possibly infinite index set) function on $\mathrm{Ob}\,\nu$. Then define function f' on $\mathrm{SLA}(\mathrm{Ob}\,\nu)$ as:

$$f'(b) = \left\{ g \circ \prod^{(A)} b \mid g \in f' \right\}.$$

We can't use cross-composition product instead of above sub-atomic product because cross-composition product is not a funcoid (just a pointfree funcoid). We can replace sub-atomic product with displaced product, but as about my opinion displaced product seems more weird an inconvenient.

The above induces a trivial definition of functions on MSN but only for functions of finite arity (because having a finite set of MSN we can raise them to the same (maximum) level).

On differential equations

Replacing limit in the definition of derivative with the above defined reduced limit, the base set $\mathrm{Ob}\,\mu$ with MSN and operations f on the set $\mathrm{Ob}\,\mu$ with corresponding operations on MSN, we get a new interpretation of a differential equation (DE) (ordinary or partial).

Let call such (enhanced) differential equations *meta-singular equations* (as opposed to *non-singular equations* that is customary differential equations).

There arise the following questions:

Definition 30. I call a solution of a DE a *trivial restriction* if it is a restriction (to the set of non-singular points) of exactly one enhanced DE.

We need to find when there are solutions which are not trivial restrictions.

Then we can split such non-trivial solutions into following classes:

- "added solutions" are solutions whose restriction to non-singularity points is not a non-singular solution;