Proof. ?? □

Galufuncoids

Let $\mathfrak{A}$ and $\mathfrak{B}$ are **Rel**-morphisms. I will denote like $(\sim_{\mathfrak{A}}) = \operatorname{GR}\mathfrak{A}$ and $(\sim_{\mathfrak{B}}) = \operatorname{GR}\mathfrak{B}$.

Definition 9. *Galufuncoids* between $\mathfrak{A}$ and $\mathfrak{B}$ is a quadruple $(\mathfrak{A}; \mathfrak{B}; \alpha; \beta)$ such that

$$\forall x \in \operatorname{Ob}\mathfrak{A}, y \in \operatorname{Ob}\mathfrak{B}: (\alpha x \sim_{\mathfrak{B}} y \Leftrightarrow x \sim_{\mathfrak{A}} \beta y).$$

Definition 10. $x\,[f]\,y \Leftrightarrow x \sim_{\operatorname{Src} f} \beta y$.

Obvious 11. $x\,[f]\,y \Leftrightarrow x \sim_{\operatorname{Src} f} \beta y \Leftrightarrow \alpha x \sim_{\operatorname{Dst} f} y$.

Remark 12. Galufuncoids are a generalization of both (pointfree) funcoids and Galois connections.

Definition 13. The *reverse* galufuncoid is defined by the formula:

$$(\mathfrak{A}; \mathfrak{B}; \alpha; \beta)^{-1} = (\mathfrak{B}; \mathfrak{A}; \beta; \alpha).$$

Proposition 14. Composition of (composable) galufuncoids is a galufuncoid.

Proof. $(\alpha_2 \circ \alpha_1) x \sim y \Leftrightarrow \alpha_2\, \alpha_1\, x \sim y \Leftrightarrow \alpha_1\, x \sim \beta_2\, y \Leftrightarrow x \sim \beta_1\, \beta_2\, y \Leftrightarrow x \sim (\beta_1 \circ \beta_2) y$. □

Obvious 15. Galufuncoids form a category (similarly to the category of pointfree funcoids).

Definition 16. On the set of galufuncoids is defined a preorder by the formula: $f \sqsubseteq g \Leftrightarrow [f] \subseteq [g]$.

Galufuncoidal product

Functional galufuncoid

Definition 17. *Functional galufuncoid* ν / Δ of ν through filter Δ is the endo-galufuncoid defined by the formulas:

$\operatorname{Ob}(\nu / \Delta) = \mathsf{FCD}(\operatorname{Base}(\Delta); \operatorname{Ob}\nu)$

$\langle \nu / \Delta \rangle f = \nu \circ f$ and $\langle (\nu / \Delta)^{-1} \rangle f = \nu^{-1} \circ f$

$f \sim_{\operatorname{Ob}(\nu / \Delta)} g \Leftrightarrow g^{-1} \circ f \sqsupseteq \operatorname{id}^{\mathsf{FCD}}_{\Delta}$

[TODO: Restrict to the special case $f = \nu \circ F$ to make it T_2.]

[TODO: $X\,[\operatorname{SLA}(f)]\,Y$ is defined as existence of $x \in X$ such that for every entourage of x there is $y \in Y$ which is a subfilter of this entourage.]

Obvious 18. $\sim_{\operatorname{Ob}(\nu / \Delta)}$ is a symmetric relation.

Proposition 19. This is really a galufuncoid and $f\,[\nu / \Delta]\,g \Leftrightarrow g^{-1} \circ \nu \circ f \sqsupseteq \operatorname{id}_{\Delta}$.