$\forall a\in\operatorname{StarHom}(\lambda i\in\operatorname{dom}x:\operatorname{Src}x_i)$: $\operatorname{GR}\operatorname{StarComp}(a;x)=\emptyset\Rightarrow\operatorname{GR}\operatorname{StarComp}\left(\left(\operatorname{dom}x;\prod_{i\in\operatorname{dom}x}\operatorname{Src}x_i\right);x\right)=\emptyset\Rightarrow\nexists L\in\mho^{\operatorname{arity}a}\exists y\in\prod_{i\in\operatorname{dom}x}\operatorname{Src}x_i\cap\prod_{i\in\operatorname{dom}x}\operatorname{atoms}\operatorname{Src}x_i\forall i\in\operatorname{arity}a: y_i\ [x_i]\ L_i\Leftrightarrow\nexists L\in\mho^{\operatorname{arity}a}\exists y\in\prod_{i\in\operatorname{dom}x}\operatorname{atoms}\operatorname{Src}x_i\forall i\in\operatorname{arity}a: y_i\ [x_i]\ L_i\Leftrightarrow\neg\forall i\in\operatorname{arity}a\exists L\in\mho, y\in\operatorname{atoms}\operatorname{Src}x_i: y\,[x_i]\,L\Rightarrow\neg\forall i\in\operatorname{arity}a: x_i\neq 0\Leftrightarrow x\in\mathrm{ZC}_0$.

Thus $x\in\mathrm{ZC}_0\Leftrightarrow\prod^{(C)}x=Z_1\left(\rho_1\prod^{(C)}x\right)$.

If $\rho_0\circ x=\rho_0\circ y$ then $\operatorname{arity}x=\operatorname{arity}y=n$ for some index set n.

$\rho_0\circ x=\rho_0\circ y\Rightarrow\lambda i\in n:(\operatorname{Src}x=\operatorname{Src}y\wedge\operatorname{Dst}x=\operatorname{Dst}y)\Rightarrow\rho_1\prod^{(C)}x=\mathsf{FCD}(\operatorname{StarHom}(\lambda i\in\operatorname{dom}x:\operatorname{Src}x_i);\operatorname{StarHom}(\lambda i\in\operatorname{dom}x:\operatorname{Dst}x_i))=\mathsf{FCD}(\operatorname{StarHom}(\lambda i\in\operatorname{dom}y:\operatorname{Src}y_i);\operatorname{StarHom}(\lambda i\in\operatorname{dom}y:\operatorname{Dst}y_i))=\rho_1\prod^{(C)}y$.

We have proved that it is a pre-quasi-cartesian function.

Next prove that it is a quasi-cartesian function, that is

$$\left(\prod^{(C)}\right)|_{\{x\in X_0^{\operatorname{dom}\mathfrak{A}}\mid\rho_0\circ x=\mathfrak{A}\}\setminus\mathrm{ZC}_0}$$

is an injection for every indexed family $\mathfrak{A}$ of forms. Let $x\in\{x\in X_0^{\operatorname{dom}\mathfrak{A}}\mid\rho_0\circ x=\mathfrak{A}\}\setminus\mathrm{ZC}_0$. To prove that it is an injection we will restore the value of x from $\prod^{(C)}x$.

$\left\langle\prod^{(C)}x\right\rangle p=\operatorname{StarComp}(p;x)$ for every $p\in\prod_{i\in n}\operatorname{atoms}\operatorname{Src}x_i$.

It is easy to see that $\operatorname{GR}p\cap\prod_{i\in n}\operatorname{atoms}\operatorname{Src}x_i=\{p\}$. Thus

$L\in\operatorname{GR}\operatorname{StarComp}(p;x)\Leftrightarrow\forall i\in n: p_i\,[x_i]\,L_i\Leftrightarrow\forall i\in n: L_i\in\langle x_i\rangle p_i$ for every $L\in\prod_{i\in n}\operatorname{Src}x_i$.

Thus $\operatorname{GR}\operatorname{StarComp}(p;x)=\prod_{i\in n}\langle x_i\rangle p_i$.

Since $x_i\neq 0$ there exist p such that $\langle x_i\rangle p_i\neq 0$. Take $k\in n$, $p'_i=p_i$ for $i\neq k$ and $p'_k=q$ for an arbitrary value q; then

$$\langle x_k\rangle q=\Pr_k\prod_{i\in n}\langle x_i\rangle p'_i=\Pr_k\operatorname{GR}\operatorname{StarComp}(p';x)=\Pr_k\operatorname{GR}\left\langle\prod^{(C)}x\right\rangle p'. \tag{2}$$

Note that the theorem ?? in [?] applies to every x_i.

So the value of x can be restored from $\prod^{(C)}x$ by this formula.

It remained to prove that it is with injective aggregation.

We have $\Upsilon F=(\operatorname{StarHom}(\lambda i\in\operatorname{dom}f: F_{i,0});\operatorname{StarHom}(\lambda i\in\operatorname{dom}f: F_{i,1}))$ for every form F.

It is really an injection because $\operatorname{StarHom}(-)$ are disjoint. □

Conjecture 38. *Cross-composition product (for small indexed families of reloids) is a quasi-cartesian function (with injective aggregation) from the quasi-cartesian situation $\mathfrak{S}_0$ of reloids to the quasi-cartesian situation $\mathfrak{S}_1$ of pointfree funcoids over posets with least elements.*

Remark 39. The above conjecture is unsolved even for product of two multipliers.

Theorem 40. *Reloidal product (for small indexed families of filters on powersets) with multireloid projections is a product-projection system with injective aggregation from the quasi-cartesian situation $\mathfrak{S}_0$ of filters to the quasi-cartesian situation $\mathfrak{S}_1$ of multireloids.*