2. $\rho\circ x=\rho\circ y\Rightarrow\rho\prod^{(\mathrm{ord})}x=\rho\prod^{(\mathrm{ord})}y$;

that is

1. $x\in\mathrm{ZC}\Leftrightarrow\prod^{(\mathrm{ord})}x=Z\left(\operatorname{arity}\prod^{(\mathrm{ord})}x\right)$;

2. $\operatorname{arity}\circ x=\operatorname{arity}\circ y\Rightarrow\operatorname{arity}\prod^{(\mathrm{ord})}x=\operatorname{arity}\prod^{(\mathrm{ord})}y$;

that is

1. $x\in\mathrm{ZC}\Leftrightarrow\prod^{(\mathrm{ord})}x=Z\left(\operatorname{arity}\prod^{(\mathrm{ord})}x\right)$;

2. $\operatorname{arity}\circ x=\operatorname{arity}\circ y\Rightarrow\sum(\operatorname{arity}\circ x)=\sum(\operatorname{arity}\circ y)$;

but these formulas are obvious.

Next prove that it is a quasi-cartesian function. We need to show that for every indexed family of sets

$$\left(\prod^{(D)}x\right)|_{\{x\in X^{\operatorname{dom}\mathfrak{A}}\mid\rho\circ x=\mathfrak{A}\}\setminus\mathrm{ZC}}$$

is injection. This follows from the known fact that $(\prod x)|_{\{x\in X^{\operatorname{dom}\mathfrak{A}}\mid\rho\circ x=\mathfrak{A}\}\setminus\mathrm{ZC}}$ is an injection. [TODO: More detailed proof.] □

Definition 30. *The quasi-cartesian situation of pointfree funcoids over posets with least elements is:*

1. *Forms are pairs* $(\mathfrak{A};\mathfrak{B})$ *of posets with least elements.*

2. *Arguments are pointfree funcoids.*

3. *The form of an argument* f *is* $(\operatorname{Src}f;\operatorname{Dst}f)$.

4. *Zero of the form* $(\mathfrak{A};\mathfrak{B})$ *is* $0^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}=(\mathfrak{A}\times\{0^{\mathfrak{B}}\};\mathfrak{B}\times\{0^{\mathfrak{A}}\})$. *(It exists because* $\mathfrak{A}$ *and* $\mathfrak{B}$ *have least elements.)*

Proposition 31. *It is really a quasi-cartesian situation.*

Proof. We need to prove $\rho\circ Z\circ\rho=\rho$. Really,

$$\rho Z\rho f=\rho Z(\operatorname{Src}f;\operatorname{Dst}f)=\rho 0^{\mathsf{FCD}(\operatorname{Src}f;\operatorname{Dst}f)}=(\operatorname{Src}f;\operatorname{Dst}f)=\rho f.$$ □

Definition 32. *The quasi-cartesian situation of binary relations is:*

1. *Forms are pairs* $(A;B)$ *of sets.*

2. *Arguments are* **Rel***-morphisms;*

3. *The form of an argument* f *is* $(\operatorname{Src}f;\operatorname{Dst}f)$.

4. *Zero of the form* $(A;B)$ *is the* **Rel***-morphism* $(\emptyset;A;B)$.