- *Zero Z for a form is the empty relation of that form.*

Proposition 27. *The quasi-cartesian situation of anchored relations is really a quasi-cartesian situation.*

Proof. We need to prove $\rho \circ Z \circ \rho = \rho$. Really let f is an anchored relation of the form $\mathfrak{A}$. Then $Z\rho f$ is the zero relation of the same form ρf. Consequently $\rho Z \rho f = \rho f$. □

Proposition 28. *Reindexation product (for small indexed families of relation) is a quasi-cartesian function with injective aggregation from the quasi-cartesian situation of anchored relations to the same quasi-cartesian situation.*

Proof. First prove that it is a pre-quasi-cartesian function. We need to show that for every small indexed families x, y of anchored relations:

1. $x \in \mathrm{ZC} \Leftrightarrow \prod^{(D)} x = Z\left(\rho \prod^{(D)} x\right)$;
2. $\rho \circ x = \rho \circ y \Rightarrow \rho \prod^{(D)} x = \rho \prod^{(D)} y$;

that is

1. $x \in \mathrm{ZC} \Leftrightarrow \prod^{(D)} x = Z\left(\mathrm{arity} \prod^{(D)} x\right)$;
2. $\mathrm{arity} \circ x = \mathrm{arity} \circ y \Rightarrow \mathrm{arity} \prod^{(D)} x = \mathrm{arity} \prod^{(D)} y$;

that is

1. $x \in \mathrm{ZC} \Leftrightarrow \prod^{(D)} x = Z\left(\mathrm{arity} \prod^{(D)} x\right)$;
2. $\mathrm{arity} \circ x = \mathrm{arity} \circ y \Rightarrow \mathrm{uncurry}(\mathrm{arity} \circ x) = \mathrm{uncurry}(\mathrm{arity} \circ y)$;

but these formulas are obvious.

Next prove that it is a quasi-cartesian function. We need to show that for every indexed family of sets

$$\left(\prod^{(D)} x\right)|_{\{x \in X^{\mathrm{dom}\,\mathfrak{A}} \mid \rho \circ x = \mathfrak{A}\} \setminus \mathrm{ZC}}$$

is injection. This follows from the known fact that $(\prod x)|_{\{x \in X^{\mathrm{dom}\,\mathfrak{A}} \mid \rho \circ x = \mathfrak{A}\} \setminus \mathrm{ZC}}$ is an injection.

Last, we need to prove that it is with injective aggregation. Define $\Upsilon(\rho \circ x) = \rho \prod^{(D)} x$ that is $\Upsilon(\mathrm{arity} \circ x) = \mathrm{uncurry}(\mathrm{arity} \circ x)$ that is $\Upsilon p = \mathrm{uncurry}\, p$. Obviously this Υ is injective. □

Proposition 29. *Ordinated product (for small indexed families of relation) is a quasi-cartesian function from the quasi-cartesian situation of anchored relations to the same quasi-cartesian situation.*

Proof. First prove that it is a pre-quasi-cartesian function. We need to show that for every small indexed families x, y of anchored relations:

1. $x \in \mathrm{ZC} \Leftrightarrow \prod^{(\mathrm{ord})} x = Z\left(\rho \prod^{(\mathrm{ord})} x\right)$;