There exists a function Υ (*aggregation*) conforming to the formula $\Upsilon(\rho_0 \circ x) = \rho_1 fx$.

A pre-quasi-cartesian function can be described first defining a function Υ from small indexed families of forms into forms such that $\rho_1 fx = \Upsilon(\rho_0 \circ x)$ and $x \in \mathrm{ZC}_0 \Leftrightarrow fx = Z_1 \Upsilon(\rho_0 \circ x)$.

Definition 5. *A quasi-cartesian function is such pre-quasi-cartesian function f that $f|_{\{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A}\} \setminus \mathrm{ZC}_0}$ is an injection for every indexed family $\mathfrak{A}$ of forms.*

Definition 6. *A pre-quasi-cartesian function with injective aggregation is a pre-quasi-cartesian function for which the Υ function is injective.*

Exercise 1. Prove that the above defined "cartesian product of an indexed family of sets" is a quasi-cartesian function for two quasi-cartesian systems with injective aggregation.

Definition 7. *Restriction of a quasi-cartesian situation is this quasi-cartesian situation with the set of arguments X replaced by a smaller set X' such that* $\operatorname{im} Z \subseteq X'$ *and forms of arguments ρ replaced with $\rho' = \rho|_{X'}$.*

Proposition 8. *Every restriction of a quasi-cartesian situation is a quasi-cartesian situation.*

Proof. We need to prove $\rho' \circ Z \circ \rho' = \rho'$. This formula follows from $\rho' = \rho|_X$ and $\operatorname{dom} \rho' \supseteq \operatorname{im} Z$. □

Definition 9. *Restriction of a pre-quasi-cartesian function is the restriction of the source quasi-cartesian situation, the destination quasi-cartesian situation, together with a restriction of the quasi-cartesian function to indexed families of the new set of (source) arguments.*

Obvious 10. *Restriction of a pre-quasi-cartesian function is a pre-quasi-cartesian function.*

Obvious 11. *Restriction of a quasi-cartesian function is a quasi-cartesian function.*

Obvious 12. *Restriction of a (pre-)quasi-cartesian situation with injective aggregation is a (pre-)quasi-cartesian situation with injective aggregation.*

When $\operatorname{card} \langle f \rangle \{x\} = 1$ for a binary relation f, we will denote $f(x)$ or fx the element of the singleton $\langle f \rangle \{x\}$.

Proposition 13. *For pre-quasi-cartesian function f we have*

$$(\langle f \rangle \{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A}\}) \setminus \{Z_1(\Upsilon_f \mathfrak{A})\} = \langle f \rangle (\{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho \circ x = \mathfrak{A}\} \setminus \mathrm{ZC}_1).$$

Proof.

$$\begin{aligned} \langle f \rangle (\{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A}\} \setminus \mathrm{ZC}_0) &= \\ \langle f \rangle \{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A} \wedge x \notin \mathrm{ZC}_0\} &= \\ \langle f \rangle \{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A} \wedge fx \neq Z_1 \Upsilon_f (\rho_0 \circ x)\} &= \\ \langle f \rangle \{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A} \wedge fx \neq Z_1 \Upsilon_f \mathfrak{A}\} &= \\ (\langle f \rangle \{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A}\}) \setminus \{Z_1(\Upsilon_f \mathfrak{A})\}. & \end{aligned}$$

□

Proposition 14. $\operatorname{card} \langle f^{-1} \rangle \{y\} = 1$ *if* $y \in (\langle f \rangle \{x \in X_0^{\mathrm{dom}\,\mathfrak{A}} \mid \rho_0 \circ x = \mathfrak{A}\}) \setminus \{Z_1(\Upsilon_f \mathfrak{A})\}$.