6.3 Cross-composition of pointfree funcoids

[TODO: This section is partially written.]

Let now $\mathcal{C}$ be the category of all small pointfree funcoids. Let principal morphisms be principal funcoids.

Define $\pi_i^X = \mathrm{Pr}_i\ (\mathsf{RLD})_{\mathrm{in}}\ X$ whenever X is a funcoid. [TODO: We should generalize it for multifuncoids or staroids.]

7 Initial and terminal objects

Initial object of Fcd is the endofuncoid $\uparrow^{\mathsf{FCD}(\emptyset;\emptyset)}\emptyset$. It is initial because it has precisely one morphism o (the empty set considered as a function) to any object Y. o is a morphism because $o\circ\uparrow^{\mathsf{FCD}(\emptyset;\emptyset)}\emptyset\sqsubseteq Y\circ o$.

Proposition 52. Terminal objects of Fcd are exactly $\uparrow^{\mathfrak{F}}\{*\}\times^{\mathsf{FCD}}\uparrow^{\mathfrak{F}}\{*\}=\uparrow^{\mathsf{FCD}}\{(*;*)\}$ where $*$ is an arbitrary point.

Proof. In order for a function $f: X\to\uparrow^{\mathsf{FCD}}\{(*;*)\}$ be a morphism, it is required exactly $f\circ X\sqsubseteq\uparrow^{\mathsf{FCD}}\{(*;*)\}\circ f$

$f\circ X\sqsubseteq(f^{-1}\circ\uparrow^{\mathsf{FCD}}\{(*;*)\})^{-1}$; $f\circ X\sqsubseteq(\{*\}\times^{\mathsf{FCD}}\langle f^{-1}\rangle\{*\})^{-1}$; $f\circ X\sqsubseteq\langle f^{-1}\rangle\{*\}\times^{\mathsf{FCD}}\{*\}$ what true exactly when f is a constant function with the value $*$. □

If $n=\emptyset$ then $Z=\{\emptyset\}$; $\prod^{(L)}\emptyset=\max\mathsf{FCD}(Z;Z)=\uparrow^{\mathfrak{F}}\{\emptyset\}\times^{\mathsf{FCD}}\uparrow^{\mathfrak{F}}\{\emptyset\}=\uparrow^{\mathsf{FCD}}\{(\emptyset;\emptyset)\}$.

[TODO: Initial and terminal objects of Rld.]

8 Canonical product and subatomic product

[TODO: Confusion between filters on products and multireloids.]

Proposition 53. $\mathrm{Pr}_i^{\mathsf{RLD}}|_{\mathfrak{F}(Z)}=\langle\pi_i\rangle$ for every index i of a cartesian product Z.

Proof. If $\mathcal{X}\in\mathfrak{F}(Z)$ then $(\mathrm{Pr}_i^{\mathsf{RLD}}|_{\mathfrak{F}(Z)})\mathcal{X}=\mathrm{Pr}_i^{\mathsf{RLD}}\mathcal{X}=\bigsqcap\langle\uparrow\rangle\langle\mathrm{Pr}_i\rangle\mathcal{X}=\bigsqcap\langle\pi_i\rangle\mathrm{up}\,\mathcal{X}=\langle\pi_i\rangle\mathcal{X}$. □

Proposition 54. $\prod^{(A)}F=\bigsqcap_{i\in n}\left(\left(\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Dst}\,F)}\right)^{-1}\circ F_i\circ\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Src}\,F)}\right)$.

Proof. $a\left[\prod^{(A)}F\right]b\Leftrightarrow\forall i\in\mathrm{dom}\,F:\mathrm{Pr}_i^{\mathsf{RLD}}a\,[F_i]\,\mathrm{Pr}_i^{\mathsf{RLD}}b\Leftrightarrow\forall i\in\mathrm{dom}\,F:\left\langle\left(\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Dst}\,F)}\right)^{-1}\right\rangle[F_i]\left\langle\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Src}\,F)}\right\rangle\Leftrightarrow\forall i\in\mathrm{dom}\,F: a\left[\left(\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Dst}\,F)}\right)^{-1}\circ F_i\circ\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Src}\,F)}\right]b\Leftrightarrow a\left[\bigsqcap_{i\in n}\left(\left(\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Dst}\,F)}\right)^{-1}\circ F_i\circ\pi_i^{\mathsf{FCD}(\prod_{i\in n}\mathrm{Src}\,F)}\right)\right]b$ for ultrafilters a and b. □

Corollary 55. $\prod^{(L)}F=\prod^{(A)}F$ is F is a small indexed family of funcoids.

9 Further plans

Does the formula $\prod^{(L)}_{i\in n}(g_i\circ f_i)=\prod^{(L)}g\circ\prod^{(L)}f$ hold?

Conjecture 56. The categories Fcd and Rld are cartesian closed (actually two conjectures).

http://mathoverflow.net/questions/141615/how-to-prove-that-there-are-no-exponential-object-in-a-category suggests to investigate colimits to prove that there are no exponential object.

Coordinate-wise continuity.

Bibliography

[1] Victor Porton. *Algebraic General Topology. Volume 1*. 2013.