Remark 33. Thus

$$F_0 \amalg^{(L)} F_1 = \left(\iota_0^{(\mathrm{Src}\, F_0; \mathrm{Src}\, F_1)} \circ F_0^\dagger \circ \left(\iota_0^{(\mathrm{Dst}\, F_0; \mathrm{Dst}\, F_1)}\right)^\dagger\right) \sqcup \left(\iota_1^{(\mathrm{Src}\, F_0; \mathrm{Src}\, F_1)} \circ F_1^\dagger \circ \left(\iota_1^{(\mathrm{Dst}\, F_0; \mathrm{Dst}\, F_1)}\right)^\dagger\right)$$

that is coproduct is defined by a pure algebraic formula.

Proposition 34. $\coprod^{(L)} F = \min\left\{\Phi \in \mathrm{End}\left(\coprod_{j\in n}^{(Q)} \mathrm{Ob}\, F_j\right) \mid \forall i \in n: \Phi \sqsupseteq \iota_i^{\lambda j\in n:\mathrm{Src}\, F_j} \circ F_i^\dagger \circ \left(\iota_i^{\lambda j\in n:\mathrm{Dst}\, F_j}\right)^\dagger\right\}$.

Proof. By definition of meet on a complete lattice. □

Corollary 35. $\coprod^{(L)} F = \bigsqcap\left\{\Phi \in \mathrm{End}\left(\coprod_{j\in n}^{(Q)} \mathrm{Ob}\, F_j\right) \mid \forall i \in n: \Phi \sqsupseteq \iota_i^{\lambda j\in n:\mathrm{Src}\, F_j} \circ F_i^\dagger \circ \left(\iota_i^{\lambda j\in n:\mathrm{Dst}\, F_j}\right)^\dagger\right\}$.

Theorem 36. Let π_i^X be metainjective morphisms. If $S \in \mathscr{P}(\mathrm{Mor}(A_0; B_0) \times \mathrm{Mor}(A_1; B_1))$ for some sets A_0, B_0, A_1, B_1 then

$$\bigsqcup \left\{a \times^{(L)} b \mid (a; b) \in S\right\} = \bigsqcup \mathrm{dom}\, S \times^{(L)} \bigsqcup \mathrm{im}\, S.$$

Corollary 37. $\left(a_0 \amalg^{(L)} b_0\right) \sqcup \left(a_1 \amalg^{(L)} b_1\right) = (a_0 \sqcap a_1) \amalg^{(L)} (b_0 \sqcap b_1)$.

Corollary 38. $a_0 \amalg^{(L)} b_0 \equiv a_1 \amalg^{(L)} b_1 \Leftrightarrow a_0 \equiv a_1 \wedge b_0 \equiv b_1$.

5.2 Supremum coproduct for endomorphisms

Let F be an indexed family of endomorphisms of $\mathcal{C}$.

I will denote $\mathrm{Ob}\, f$ the object (source and destination) of an endomorphism f.

Let also ι_i be a monovalued entirely defined morphism (for each $i \in \mathrm{dom}\, F$).

Definition 39. $\coprod^{(L)} F = \bigsqcup_{i\in \mathrm{dom}\, F} \left(\iota_i^{\lambda j\in n:\mathrm{Ob}\, F_j} \circ F_i^\dagger \circ \left(\iota_i^{\lambda j\in n:\mathrm{Ob}\, F_j}\right)^\dagger\right)$ (if ι is defined at $\lambda j \in n: \mathrm{Ob}\, F_j$). (I call it *supremum coproduct*).

Abbreviate $\iota_i = \iota_i^{\lambda j\in n:\mathrm{Ob}\, F_j}$.

So $\coprod F = \bigsqcup_{i\in \mathrm{dom}\, F} \left(\iota_i \circ F_i^\dagger \circ (\iota_i)^\dagger\right)$.

$\coprod F = \min\left\{\Phi \in \mathrm{End}\left(\coprod_{j\in n}^{(Q)} \mathrm{Ob}\, F_j\right) \mid \forall i \in n: \Phi \sqsupseteq \iota_i \circ F_i^\dagger \circ (\iota_i)^\dagger\right\}$.

Taking into account that ι_i is a monovalued entirely defined morphism, we get:

Obvious 40. $\coprod^{(L)} = \min\left\{\Phi \in \mathrm{End}\left(\coprod_{j\in n}^{(Q)} \mathrm{Ob}\, F_j\right) \mid \forall i \in n: \iota_i \in \mathrm{C}\left(F_i^\dagger; \Phi\right)\right\}$.

Corollary 41. $\iota_i \in \mathrm{C}\left(F_i; \coprod^{(L)} F\right)$ for every $i \in \mathrm{dom}\, F$.

5.3 Category of continuous morphisms

[TODO: What is X?]

Let ι_i (for $i \in \mathrm{dom}\, F$) be entirely defined monovalued and metacomplete morphisms.

Let $\bigoplus$ of an indexed family of morphisms is a morphism; $\left(\bigoplus f\right) \circ \iota_i = f_i$; $\bigoplus_{i\in n} (f \circ \iota_i) = f$ (a dual of the above).

Let $F_i \in \mathrm{End}\left(\coprod_{j\in n}^{(Q)} \mathrm{Ob}\, F_j\right)$ for all $i \in n$ (where n is some index set) (a self-dual of the above).

Definition 42. $\iota_i^{\mathrm{cont}(\mathcal{C})} = \left(\coprod^{(L)} F; F_i^\dagger; \iota_i\right)$.

Proposition 43. ι_i are continuous, that is $\iota_i^{\mathrm{cont}(\mathcal{C})}$ are morphisms.