Funcoidal product

The funcoidal product of filters is a generalization of the Cartesian product of sets.

Let $\mathcal{A}$ and $\mathcal{B}$ be filters. Then there exists a unique funcoid (the *funcoidal product* of $\mathcal{A}$ and $\mathcal{B}$) $\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ such that

$$\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle \mathcal{X} = \begin{cases} \mathcal{B} & \text{if } \mathcal{X} \sqcap \mathcal{A} \neq 0^{\mathrm{Base}(\mathcal{A})}; \\ 0^{\mathrm{Base}(\mathcal{B})} & \text{if } \mathcal{X} \sqcap \mathcal{A} = 0^{\mathrm{Base}(\mathcal{A})}; \end{cases}$$

$$\mathcal{X} \left[\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right] \mathcal{Y} \Leftrightarrow \mathcal{X} \sqcap \mathcal{A} \neq 0^{\mathrm{Base}(\mathcal{A})} \wedge \mathcal{Y} \sqcap \mathcal{B} \neq 0^{\mathrm{Base}(\mathcal{B})}.$$