

Functors as binary relations

A binary relation $\delta \in \mathcal{P}(\mathcal{P}A \times \mathcal{P}B)$ corresponds to a functor if and only if it complies to the formulas (for all suitable sets I, J, K):

$$\neg(\emptyset \delta I); \quad I \cup J \delta K \Leftrightarrow I \delta K \vee J \delta K;$$

$$\neg(I \delta \emptyset); \quad K \delta I \cup J \Leftrightarrow K \delta I \vee K \delta J.$$

The functor f and relation δ are related by the formulas:

$$\mathcal{X} [f] \mathcal{Y} \Leftrightarrow \forall X \in \mathcal{X}, Y \in \mathcal{Y}: X \delta Y;$$

$$X \delta Y \Leftrightarrow X [f]^* Y.$$