Proof. Let $f \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ and $f \neq 0^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$. Then $\operatorname{dom} f \neq 0^{\mathfrak{A}}$, thus exists $a \in \operatorname{atoms}^{\mathfrak{A}} \operatorname{dom} f$. So $\langle f \rangle a \neq 0^{\mathfrak{B}}$ thus exists $b \in \operatorname{atoms}^{\mathfrak{B}} \langle f \rangle a$. Finally the atomic pointfree funcoid $a \times^{\mathsf{FCD}} b \subseteq f$. □

Theorem 71. Let $\mathfrak{A}$, $\mathfrak{B}$ are sets of filters over boolean lattices. Then the poset $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is separable.

Proof. Let $f, g \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$, $f \subset g$. Then taking in account the theorem 51 exists $a \in \operatorname{atoms}^{\mathfrak{A}}$ such that $\langle f \rangle a \subset \langle g \rangle a$. By corollary 17 in [3] $\mathfrak{B}$ is atomically separable. So exists $b \in \operatorname{atoms}^{\mathfrak{B}}$ such that $\langle f \rangle a \cap^{\mathfrak{B}} b = 0^{\mathfrak{B}}$ and $b \subseteq \langle g \rangle a$. For every $x \in \operatorname{atoms}^{\mathfrak{A}}$

$$\langle f \rangle a \cap^{\mathfrak{B}} \langle a \times^{\mathsf{FCD}} b \rangle a = \langle f \rangle a \cap^{\mathfrak{B}} b = 0^{\mathfrak{B}},$$
$$x \neq a \Rightarrow \langle f \rangle x \cap^{\mathfrak{B}} \langle a \times^{\mathsf{FCD}} b \rangle x = \langle f \rangle x \cap^{\mathfrak{B}} 0^{\mathfrak{B}} = 0^{\mathfrak{B}}.$$

Thus $\langle f \rangle x \cap^{\mathfrak{B}} \langle a \times b \rangle x = 0^{\mathfrak{B}}$ and consequently $f \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} (a \times^{\mathsf{FCD}} b) = 0^{\mathfrak{B}}$.

$$\langle a \times^{\mathsf{FCD}} b \rangle a = b \subseteq \langle g \rangle a,$$
$$x \neq a \Rightarrow \langle a \times^{\mathsf{FCD}} b \rangle x = 0^{\mathfrak{B}} \subseteq \langle g \rangle a.$$

Thus $\langle a \times^{\mathsf{FCD}} b \rangle x \subseteq \langle g \rangle x$ and consequently $a \times^{\mathsf{FCD}} b \subseteq g$.

So the lattice of funcoids is separable by the theorem 19 in [3]. □

Corollary 72. Let $\mathfrak{A}$, $\mathfrak{B}$ are sets of filters over boolean lattices. The poset $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is:

1. separable;
2. atomically separable;
3. conforming to Wallman's disjunction property.

Proof. By the theorem 22 in [3]. □

Remark 73. For more ways to characterize (atomic) separability of the lattice of pointfree funcoids see [3], subsections "Separation subsets and full stars" and "Atomically separable lattices".

Corollary 74. Let $(\mathfrak{A};\mathfrak{Z}_0)$ and $(\mathfrak{B};\mathfrak{Z}_1)$ are primary filtrators over boolean lattices. The poset $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is an atomistic lattice.

Proof. By the theorem 30 $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is a complete lattice. Let $f \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$. Suppose contrary to the statement to be proved that $\bigcup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \operatorname{atoms}^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} f \subset f$. Then exists $a \in \operatorname{atoms}^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} f$ such that $a \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \bigcup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \operatorname{atoms}^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} f = 0^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$ what is impossible. □

Proposition 75. Let $\mathfrak{A}$, $\mathfrak{B}$ are sets of filters over boolean lattices.

$\operatorname{atoms}^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}(f \cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} g) = \operatorname{atoms}^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} f \cup \operatorname{atoms}^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} g$ for every $f, g \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$.

Proof. $(a \times^{\mathsf{FCD}} b) \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} (f \cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} g) \neq \emptyset \Leftrightarrow a \left[f \cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} g \right] b \Leftrightarrow a [f] b \vee a [g] b \Leftrightarrow (a \times^{\mathsf{FCD}} b) \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} f \neq 0^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \vee (a \times^{\mathsf{FCD}} b) \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} g \neq 0^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$ for every $a \in \operatorname{atoms}^{\mathfrak{A}}$ and $b \in \operatorname{atoms}^{\mathfrak{B}}$ (used the corollary 63 and theorem 32). □

Theorem 76. Let $(\mathfrak{A};\mathfrak{Z}_0)$ and $(\mathfrak{B};\mathfrak{Z}_1)$ are primary filtrators over boolean lattices. For every $f, g, h \in \mathsf{FCD}(\mathfrak{A};\mathfrak{B})$, $R \in \mathscr{P}\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$:

1. $f \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} (g \cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} h) = (f \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} g) \cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} (f \cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} h)$;
2. $f \cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \bigcap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} R = \bigcap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \langle f \cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} \rangle R$.

Proof. We will take in account that the lattice of funcoids is an atomistic lattice (corollary 74). To be concise I will write atoms instead of $\operatorname{atoms}^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$ and $\cap$ and $\cup$ instead of $\cap^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$ and $\cup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})}$.

1. $\operatorname{atoms}(f \cap (g \cup h)) = \operatorname{atoms} f \cap \operatorname{atoms}(g \cup h) = \operatorname{atoms} f \cap (\operatorname{atoms} g \cup \operatorname{atoms} h) = (\operatorname{atoms} f \cap \operatorname{atoms} g) \cup (\operatorname{atoms} f \cap \operatorname{atoms} h) = \operatorname{atoms}(f \cap g) \cup \operatorname{atoms}(f \cap h) = \operatorname{atoms}((f \cap g) \cup (f \cap h))$.