$$\operatorname{dom} f = \bigcup^{\operatorname{Src} f}\{a \in \operatorname{atoms}^{\operatorname{Src} f} \mid a \not\asymp^{\operatorname{Src} f} \operatorname{dom} f\} = \bigcup^{\operatorname{Src} f}\{a \in \operatorname{atoms}^{\operatorname{Src} f} \mid \langle f\rangle a \neq 0^{\operatorname{Dst} f}\}. \qquad \square$$

Proposition 49. $\operatorname{dom} f|_a^{\mathsf{FCD}(\operatorname{Src} f)} = a \sqcap^{\operatorname{Src} f} \operatorname{dom} f$ for every pointfree funcoid f and $a \in \operatorname{Src} f$ where $\operatorname{Src} f$ is a meet-semilattice and $\operatorname{Dst} f$ has greatest element.

Proof. $\operatorname{dom} f|_a^{\mathsf{FCD}(\operatorname{Src} f)} = \operatorname{im}\left(I_a^{\mathsf{FCD}(\operatorname{Src} f)} \circ f^{-1}\right) = \left\langle I_a^{\mathsf{FCD}(\operatorname{Src} f)}\right\rangle \langle f^{-1}\rangle 1^{\operatorname{Dst} f} = a \sqcap^{\operatorname{Src} f} \langle f^{-1}\rangle 1^{\operatorname{Dst} f} = a \sqcap^{\operatorname{Src} f} \operatorname{dom} f$. $\square$

Proposition 50. For every composable pointfree funcoids f and g where the posets $\operatorname{Src} f$ and $\operatorname{Dst} f = \operatorname{Src} g$ have greatest elements:

1. If $\operatorname{im} f \supseteq \operatorname{dom} g$ then $\operatorname{im}(g \circ f) = \operatorname{im} g$.
2. If $\operatorname{im} f \subseteq \operatorname{dom} g$ then $\operatorname{dom}(g \circ f) = \operatorname{dom} g$.

Proof.

1. $\operatorname{im}(g \circ f) = \langle g \circ f\rangle 1^{\operatorname{Src} f} = \langle g\rangle\langle f\rangle 1^{\operatorname{Src} f} = \langle g\rangle \operatorname{im} f = \langle g\rangle(\operatorname{im} f \sqcap^{\operatorname{Dst} f} \operatorname{dom} g) = \langle g\rangle \operatorname{dom} g = \langle g\rangle 1^{\operatorname{Src} g} = \operatorname{im} g$.
2. $\operatorname{dom}(g \circ f) = \operatorname{im}(f^{-1} \circ g^{-1})$ what by the proved is equal to $\operatorname{im} f^{-1}$ that is $\operatorname{dom} f$. $\square$

3.7 Category of pointfree funcoids

I will define the category pfFCD of pointfree funcoids:

- The class of objects are small posets.
- The set of morphisms from $\mathfrak{A}$ to $\mathfrak{B}$ is $\mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$.
- The composition is the composition of pointfree funcoids.
- Identity morphism for an object $\mathfrak{A}$ is $(\mathfrak{A}; \mathfrak{A}; (=)|_{\mathfrak{A}}; (=)|_{\mathfrak{A}})$.

To prove that it is really a category is trivial.

The *category of funcoid triples* is defined as follows:

- Objects are pairs $(\mathfrak{A}; \mathcal{A})$ where $\mathfrak{A}$ is a small poset and $\mathcal{A} \in \mathfrak{A}$.
- The morphisms from an object $(\mathfrak{A}; \mathcal{A})$ to an object $(\mathfrak{B}; \mathcal{B})$ are tuples $(f; \mathfrak{A}; \mathfrak{B}; \mathcal{A}; \mathcal{B})$ where $f \in \mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ and $\operatorname{dom} f \sqsubseteq \mathcal{A} \wedge \operatorname{im} f \sqsubseteq \mathcal{B}$.
- The composition is defined by the formula $(g; \mathcal{B}; \mathcal{C}) \circ (f; \mathcal{A}; \mathcal{B}) = (g \circ f; \mathcal{A}; \mathcal{C})$.
- Identity morphism for an object $(\mathfrak{A}; \mathcal{A})$ is $I_{\mathcal{A}}^{\mathsf{FCD}(\mathfrak{A})}$.

To prove that it is really a category is trivial.

3.8 Specifying funcoids by functions or relations on atomic filter objects

Theorem 51. Let $\mathfrak{A}$ is an atomic poset and $(\mathfrak{B}; \mathfrak{Z}_1)$ is a primary filtrator over a boolean lattice. Then for every $f \in \mathsf{FCD}(\mathfrak{A}; \mathfrak{B})$ and $\mathcal{X} \in \mathfrak{A}$ we have

$$\langle f\rangle \mathcal{X} = \bigsqcup^{\mathfrak{B}} \langle\langle f\rangle\rangle \operatorname{atoms}^{\mathfrak{A}} \mathcal{X}.$$

Proof. For every $Y \in \mathfrak{Z}_1$ we have

$$\begin{aligned} Y \not\asymp^{\mathfrak{B}} \langle f\rangle \mathcal{X} &\Leftrightarrow \mathcal{X} \not\asymp^{\mathfrak{A}} \langle f^{-1}\rangle Y \\ &\Leftrightarrow \exists x \in \operatorname{atoms}^{\mathfrak{A}} \mathcal{X}: x \not\asymp^{\mathfrak{A}} \langle f^{-1}\rangle Y \\ &\Leftrightarrow \exists x \in \operatorname{atoms}^{\mathfrak{A}} \mathcal{X}: Y \not\asymp^{\mathfrak{B}} \langle f\rangle x. \end{aligned}$$

Thus $\partial\langle f\rangle \mathcal{X} = \bigcup \langle\partial\rangle \langle\langle f\rangle\rangle \operatorname{atoms}^{\mathfrak{A}} \mathcal{X} = \partial \bigsqcup^{\mathfrak{B}} \langle\langle f\rangle\rangle \operatorname{atoms}^{\mathfrak{A}} \mathcal{X}$ (used the theorem 46 in [3]). Consequently $\langle f\rangle \mathcal{X} = \bigsqcup^{\mathfrak{B}} \langle\langle f\rangle\rangle \operatorname{atoms}^{\mathfrak{A}} \mathcal{X}$ by the corollary 15 in [3]. $\square$