Proof. $(\mathrm{Src}\, f; \mathfrak{Z}_0)$ is a finitely meet-closed filtrator by theorem 29 in [3] and with separable core by theorem 37 and corollary 10 in [3]; thus we can apply theorem 25.

$\langle f\rangle \bigcap^{\mathrm{Src}\, f} S \subseteq \langle f\rangle X$ for every $X\in S$ and thus $\langle f\rangle \bigcap^{\mathrm{Src}\, f} S \subseteq \bigcap^{\mathrm{Dst}\, f} \langle\langle f\rangle\rangle S$.

Taking into account properties of generalized filter bases:

$$\begin{aligned}
\langle f\rangle \bigcap^{\mathrm{Src}\, f} S &= \\
\bigcap^{\mathrm{Dst}\, f} \langle\langle f\rangle\rangle \mathrm{up} \bigcap S &= \\
\bigcap^{\mathrm{Dst}\, f} \{\langle\langle f\rangle\rangle X \mid \exists \mathcal{P}\in S\colon X\in \mathrm{up}\,\mathcal{P}\} &= \\
\bigcap^{\mathrm{Dst}\, f} \{\langle f\rangle X \mid \exists \mathcal{P}\in S\colon X\in \mathrm{up}\,\mathcal{P}\} &\supseteq \\
\bigcap^{\mathrm{Dst}\, f} \{\langle f\rangle \mathcal{P} \mid \mathcal{P}\in S\} &= \\
\bigcap^{\mathrm{Dst}\, f} \langle\langle f\rangle\rangle S. &
\end{aligned}$$

□

3.4 The preorder of pointfree funcoids

The *preorder of pointfree funcoids* is defined by the formula $f\subseteq g \Leftrightarrow [f]\subseteq[g]$ for every pointfree funcoids f and g.

Remark 28. It is enough to define preorder of pointfree funcoids on every set $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ where $\mathfrak{A}$ and $\mathfrak{B}$ are posets. We do not need to compare pointfree funcoids with different sources or destinations.

Theorem 29. If $\mathfrak{A}$ and $\mathfrak{B}$ are separable posets then $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is a poset.

Proof. From the theorem 12. □

Theorem 30. Let $(\mathfrak{A};\, \mathfrak{Z}_0)$ and $(\mathfrak{B};\, \mathfrak{Z}_1)$ are primary filtrators over boolean lattices. Then for $R\in \mathscr{P}\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ and $X\in\mathfrak{Z}_0$, $Y\in\mathfrak{Z}_1$ we have:

1. $X \left[\bigcup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} R\right] Y \Leftrightarrow \exists f\in R\colon X\,[f]\,Y$;
2. $\left\langle \bigcup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} R\right\rangle X = \bigcup^{\mathfrak{B}} \{\langle f\rangle X \mid f\in R\}$.

Proof.

2. $\alpha X \overset{\mathrm{def}}{=} \bigcup^{\mathfrak{B}}\{\langle f\rangle X \mid f\in R\}$ (by corollary 8 in [3] all joins on $\mathfrak{B}$ exist). We have $\alpha 0^{\mathfrak{A}} = 0^{\mathfrak{B}}$;

$$\begin{aligned}
\alpha(I\cup^{\mathfrak{Z}_0} J) &= \bigcup^{\mathfrak{B}}\{\langle f\rangle(I\cup^{\mathfrak{Z}_0} J) \mid f\in R\}\\
&= \bigcup^{\mathfrak{B}}\{\langle f\rangle(I\cup^{\mathfrak{A}} J) \mid f\in R\}\\
&= \bigcup^{\mathfrak{B}}\{\langle f\rangle I\cup^{\mathfrak{B}} \langle f\rangle J \mid f\in R\}\\
&= \bigcup^{\mathfrak{B}}\{\langle f\rangle I \mid f\in R\}\cup^{\mathfrak{B}} \bigcup^{\mathfrak{B}}\{\langle f\rangle J \mid f\in R\}\\
&= \alpha I\cup^{\mathfrak{B}} \alpha J
\end{aligned}$$

(used the theorem 15). By the theorem 26 the function α can be continued to $\langle h\rangle$ for a $h\in\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$. Obviously

$$\forall f\in R\colon h\supseteq f. \tag{4}$$

And h is the least element of $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ for which holds the condition (4). So $h = \bigcup^{\mathsf{FCD}(\mathfrak{A};\mathfrak{B})} R$.

1. $X\ [\bigcup^{\mathsf{FCD}} R]\ Y \Leftrightarrow Y\cap^{\mathfrak{B}} \langle \bigcup^{\mathsf{FCD}} R\rangle X \neq 0^{\mathfrak{B}} \Leftrightarrow Y\cap^{\mathfrak{B}} \bigcup^{\mathfrak{B}}\{\langle f\rangle X \mid f\in R\}\neq 0^{\mathfrak{B}} \Leftrightarrow \exists f\in R\colon Y\cap^{\mathfrak{B}}\langle f\rangle X\neq 0^{\mathfrak{B}} \Leftrightarrow \exists f\in R\colon X\,[f]\,Y$ (used the theorem 40 in [3]). □

Corollary 31. If $(\mathfrak{A};\mathfrak{Z}_0)$ and $(\mathfrak{B};\mathfrak{Z}_1)$ are primary filtrators over boolean lattices then $\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$ is a complete lattice.