can be continued to the function $\langle f\rangle$ for a unique $f\in\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$;

$$\langle f\rangle\mathcal{X}=\bigcap^{\mathfrak{B}}\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X} \tag{1}$$

for every $\mathcal{X}\in\mathfrak{A}$.

2. A relation $\delta\in\mathscr{P}(\mathfrak{Z}_0\times\mathfrak{Z}_1)$ conforming to the formulas (for every I, J, $K\in\mathfrak{Z}_0$ and I', J', $K'\in\mathfrak{Z}_1$)

$$\begin{array}{ll}\neg(0^{\mathfrak{Z}_0}\,\delta\, I), & I\cup^{\mathfrak{Z}_0}J\,\delta\,K'\Leftrightarrow I\,\delta\,K'\vee J\,\delta\,K',\\ \neg(I'\,\delta\,0^{\mathfrak{Z}_1}), & K\,\delta\,I'\cup^{\mathfrak{Z}_1}J'\Leftrightarrow K\,\delta\,I'\vee K\,\delta\,J'\end{array} \tag{2}$$

can be continued to the relation $[f]$ for a unique $f\in\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$;

$$\mathcal{X}\,[f]\,\mathcal{Y}\Leftrightarrow\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X},Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y}\colon X\,\delta\,Y \tag{3}$$

for every $\mathcal{X}\in\mathfrak{A}$, $\mathcal{Y}\in\mathfrak{B}$.

Proof. Existence of no more than one such pointfree funcoids and formulas (1) and (3) follow from two previous theorems.

2. $\{Y\in\mathfrak{Z}_1\mid X\,\delta\,Y\}$ is obviously a free star for every $X\in\mathfrak{Z}_0$. By properties of filters on boolean lattices, there exist a unique filter object αX such that $\partial(\alpha X)=\{Y\in\mathfrak{Z}_1\mid X\,\delta\,Y\}$ for every $X\in\mathfrak{Z}_0$. Thus $\alpha\in\mathfrak{B}^{\mathfrak{Z}_0}$. Similarly it can be defined $\beta\in\mathfrak{A}^{\mathfrak{Z}_1}$ by the formula $\partial(\beta X)=\{X\in\mathfrak{Z}_0\mid X\,\delta\,Y\}$. Let's continue the functions α and β to $\alpha'\in\mathfrak{B}^{\mathfrak{A}}$ and $\beta'\in\mathfrak{A}^{\mathfrak{B}}$ by the formulas

$$\alpha'\mathcal{X}=\bigcap^{\mathfrak{B}}\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}\quad\text{and}\quad\beta'\mathcal{X}=\bigcap^{\mathfrak{A}}\langle\beta\rangle\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{X}$$

and δ to $\delta'\in\mathscr{P}(\mathfrak{A}\times\mathfrak{B})$ by the formula

$$\mathcal{X}\,\delta'\,\mathcal{Y}\Leftrightarrow\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X},Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y}\colon X\,\delta\,Y.$$

$\mathcal{Y}\cap^{\mathfrak{B}}\alpha'\mathcal{X}\neq0^{\mathfrak{B}}\Leftrightarrow\mathcal{Y}\cap^{\mathfrak{B}}\bigcap^{\mathfrak{B}}\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}\neq0^{\mathfrak{B}}\Leftrightarrow\bigcap^{\mathfrak{B}}\langle\mathcal{Y}\cap^{\mathfrak{B}}\rangle\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}\neq0^{\mathfrak{B}}$. Let's prove that

$$W=\langle\mathcal{Y}\cap^{\mathfrak{B}}\rangle\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$$

is a generalized filter base: To prove it is enough to show that $\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$ is a generalized filter base.

If $\mathcal{A},\mathcal{B}\in\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$ then exist $X_1,X_2\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$ such that $\mathcal{A}=\alpha X_1$ and $\mathcal{B}=\alpha X_2$.

Then $\alpha(X_1\cap^{\mathfrak{Z}_0}X_2)\in\langle\alpha\rangle\operatorname{up}\mathcal{X}$. So $\langle\alpha\rangle\operatorname{up}\mathcal{X}$ is a generalized filter base and thus W is a generalized filter base.

By properties of generalized filter bases, $\bigcap^{\mathfrak{B}}\langle\mathcal{Y}\cap^{\mathfrak{B}}\rangle\langle\alpha\rangle\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}\neq0^{\mathfrak{B}}$ is equivalent to

$$\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}\colon\mathcal{Y}\cap^{\mathfrak{B}}\alpha X\neq0^{\mathfrak{B}},$$

what is equivalent to $\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$, $Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y}$: $Y\cap^{\mathfrak{B}}\alpha X\neq0^{\mathfrak{B}}\Leftrightarrow\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$, $Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y}$: $Y\in\partial(\alpha X)\Leftrightarrow\forall X\in\operatorname{up}^{(\mathfrak{A};\mathfrak{Z}_0)}\mathcal{X}$, $Y\in\operatorname{up}^{(\mathfrak{B};\mathfrak{Z}_1)}\mathcal{Y}$: $X\,\delta\,Y$. Combining the equivalencies we get $\mathcal{Y}\cap^{\mathfrak{B}}\alpha'\mathcal{X}\neq0\Leftrightarrow X\,\delta'\,Y$. Analogously $\mathcal{X}\cap^{\mathfrak{A}}\beta'\mathcal{Y}\neq0^{\mathfrak{A}}\Leftrightarrow X\,\delta'\,Y$. So $\mathcal{Y}\cap^{\mathfrak{B}}\alpha'\mathcal{X}\neq0^{\mathfrak{B}}\Leftrightarrow\mathcal{X}\cap^{\mathfrak{A}}\beta'\mathcal{Y}\neq0^{\mathfrak{A}}$, that is $(\mathfrak{A};\mathfrak{B};\alpha';\beta')$ is a pointfree funcoid. From the formula $\mathcal{Y}\cap^{\mathfrak{B}}\alpha'\mathcal{X}\neq\emptyset\Leftrightarrow X\,\delta'\,Y$ follows that $[(\mathfrak{A};\mathfrak{B};\alpha';\beta')]$ is a continuation of δ.

1. Let define the relation $\delta\in\mathscr{P}(\mathfrak{Z}_0\times\mathfrak{Z}_1)$ by the formula $X\,\delta\,Y\Leftrightarrow Y\cap^{\mathfrak{B}}\alpha X\neq0^{\mathfrak{B}}$.

That $\neg(0^{\mathfrak{Z}_0}\,\delta\,I')$ and $\neg(I\,\delta\,0^{\mathfrak{Z}_1})$ is obvious. We have $K\,\delta\,I'\cup^{\mathfrak{Z}_1}J'\Leftrightarrow(I'\cup^{\mathfrak{Z}_1}J')\cap^{\mathfrak{B}}\alpha K\neq0^{\mathfrak{B}}\Leftrightarrow(I'\cup^{\mathfrak{B}}J')\cap^{\mathfrak{B}}\alpha K\neq0^{\mathfrak{B}}\Leftrightarrow(I'\cap^{\mathfrak{B}}\alpha K)\cup^{\mathfrak{B}}(J'\cap^{\mathfrak{B}}\alpha K)\neq0^{\mathfrak{B}}\Leftrightarrow I'\cap^{\mathfrak{B}}\alpha K\neq0^{\mathfrak{B}}\vee J'\cap^{\mathfrak{B}}\alpha K\neq0^{\mathfrak{B}}\Leftrightarrow K\,\delta\,I'\vee K\,\delta\,J'$ and $I\cup^{\mathfrak{Z}_0}J\,\delta\,K'\Leftrightarrow K'\cap^{\mathfrak{B}}\alpha(I\cup^{\mathfrak{Z}_0}J)\neq0^{\mathfrak{B}}\Leftrightarrow K'\cap^{\mathfrak{B}}(\alpha I\cup^{\mathfrak{B}}\alpha J)\neq0^{\mathfrak{B}}\Leftrightarrow(K'\cap^{\mathfrak{B}}\alpha I)\cup^{\mathfrak{B}}(K'\cap^{\mathfrak{B}}\alpha J)\neq0^{\mathfrak{B}}\Leftrightarrow K'\cap^{\mathfrak{B}}\alpha I\neq0^{\mathfrak{B}}\vee K'\cap^{\mathfrak{B}}\alpha J\neq0^{\mathfrak{B}}\Leftrightarrow I\,\delta\,K'\vee J\,\delta\,K'$.

That is the formulas (2) are true.

Accordingly the above δ can be continued to the relation $[f]$ for some $f\in\mathsf{FCD}(\mathfrak{A};\mathfrak{B})$.

$\forall X\in\mathfrak{Z}_0,Y\in\mathfrak{Z}_1\colon(Y\cap^{\mathfrak{B}}\langle f\rangle X\neq0\Leftrightarrow X\,[f]\,Y\Leftrightarrow Y\cap^{\mathfrak{B}}\alpha X\neq0^{\mathfrak{B}})$, consequently $\forall X\in\mathfrak{Z}_0\colon\alpha X=\langle f\rangle X$ because our filtrator is with separable core. So $\langle f\rangle$ is a continuation of α. □

Proposition 27. Let $(\operatorname{Src}f;\mathfrak{Z}_0)$ is a primary filtrator over a bounded distributive lattice element and $(\operatorname{Dst}f;\mathfrak{Z}_1)$ is a primary filtrator over a distributive lattice. If S is a generalized filter base on $\operatorname{Src}f$ then $\langle f\rangle\bigcap^{\operatorname{Src}f}S=\bigcap^{\operatorname{Dst}f}\langle\langle f\rangle\rangle S$ for every pointfree funcoid f.