

Definition. System of pointfree relations **induced** by binary relations is the system of pointfree relations whose precategory is the category **Rel** of binary relations, possible domains and images for **Rel**(A, B) are $\mathcal{P}A$ and $\mathcal{P}B$ correspondingly and domain and image are domain and image as usually defined for binary relations.

The pointfree relation induced by a binary relation is this binary relation considered as a pointfree binary relation.

Definition. Pointfree endo-relations are endo-relations whose source and destination (in our precategory) is the same.

Obvious 1. *System of pointfree relations induced by binary relations is really a system of pointfree relations.*

Remark. The category of pointfree relations induced by binary relations is a large category (its set of objects is a proper class).

Definition. The **isomorphism** between pointfree endo-relations μ and ν of a system of pointfree relations is an isomorphism f of the precategory of this system such that $\nu = f^{-1} \circ \mu \circ f$.

Obvious 2. *The precategory for the system of pointfree relations induced by binary relations is a category.*

Let describe reversal $f \mapsto f^{-1}$ of a binary relation f in pointfree terms:

f is a join (i.e., supremum) of atomic binary relation. Reversal of an atomic relation t is defined as the unique relation t^{-1} conforming to the formulas $\text{dom } t^{-1} = \text{im } t$, $\text{im } t^{-1} = \text{dom } t$. So f^{-1} is the supremum of such t^{-1} . Trivially combining these steps we get reversal of a binary relation described in pointfree terms.

Thus we have proved:

Proposition 1. *Reversal of binary relations can be restored, knowing only induced pointfree relations (up to isomorphism of systems of pointfree relations).*

Proposition 2. *Identity relation for binary relations can be restored, knowing only induced pointfree relations (up to isomorphism of systems of pointfree relations).*

Proof. Identity relation id is the identity element of our semigroup (and thus does not depend on the isomorphism). \square

Proposition 3. *The set of bijections for relations **Rel**(A, B) can be restored, knowing only induced pointfree relations (up to isomorphism of systems of pointfree relations).*

Proof. Bijections $f : A \rightarrow B$ are characterized by the formulas $f \circ f^{-1} = \text{id}_B$ and $f^{-1} \circ f = \text{id}_A$. \square