Pointfree binary relations

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November 24, 2015

Abstract

I define **pointfree binary relations**, a way to describe binary relations and more general structures without referring to particular "points" (elements).

In this short article I define **pointfree binary relations**, that is a way to describe binary relations and more general structures without referring to particular "points" (elements).

Definition. Remind that **binary relations** (also called morphisms of the category **Rel**) are triples (A, B, f) where A, B are sets and $f \in \mathscr{P}(A \times B)$.

Definition. Endo-relations are relations of the form (A, A, f) for some set A and $f \in \mathscr{P}(A \times A)$.

Definition. Isomorphism between relations $\mu \in \mathscr{P}(A_0 \times A_0)$ and $\nu \in \mathscr{P}(A_1 \times A_1)$ is a bijection $f : A_1 \to A_0$ such that $\nu = f^{-1} \circ \mu \circ f$.

I remind that **precategory** is defined as category without requirement of existence of identity morphisms.

Definition. Ordered precategory is a precategory, each Hom-set of which is a poset, subject to the inequalities

$$f_0 \le f_1 \wedge g_0 \le g_1 \Rightarrow g_0 \circ f_0 \le g_1 \circ f_1.$$

Definition. Isomorphism of ordered precategories is a map which is both precategory isomorphism and order isomorphism.

Definition. System of pointfree relations is:

- an ordered precategory (I call morphisms of which **pointfree (binary) relations**; the composition is denoted $\circ$);
- for each Hom-sets two posets: **possible domains** and **possible images**;
- **domain** dom and **image** im (maps from pointfree relations to possible domains and possible images of its Hom-set), subject to the inequalities

$$f \le g \Rightarrow \operatorname{dom} f \le \operatorname{dom} g \wedge \operatorname{im} f \le \operatorname{im} g.$$