Proposition 176. $\Pr_k \mathrm{GR} \prod^{\mathsf{Strd}} x = \star x_k$.

Proof. $\Pr_k \mathrm{GR} \prod^{\mathsf{Strd}} x = \Pr_k \{ L \in \mho^{\operatorname{dom} x} \mid \forall i \in \operatorname{dom} x : x_i \not\asymp L_i \} = ?? = \{ l \mid x_k \not\asymp l \} = \star x_k$. □

Proposition 177. $\Pr_k^{\mathsf{Strd}} \prod^{\mathsf{Strd}} x = x_k$ if x is an indexed family of proper filters, and $k \in \operatorname{dom} x$.

Proof. $\Pr_k^{\mathsf{Strd}} \prod^{\mathsf{Strd}} x = \left\langle \prod^{\mathsf{Strd}} x \right\rangle_k \left(\lambda i \in (\operatorname{dom} x) \setminus \{k\} : 1^{(\operatorname{form} x)_i} \right)$.

Thus $\partial \Pr_k^{\mathsf{Strd}} \prod^{\mathsf{Strd}} x = \left(\operatorname{val} \prod^{\mathsf{Strd}} x \right)_k \left(\lambda i \in (\operatorname{dom} x) \setminus \{k\} : 1^{(\operatorname{form} x)_i} \right) = \left\{ X \in \left(\operatorname{form} \prod^{\mathsf{Strd}} x \right)_k \mid \left(\lambda i \in (\operatorname{dom} x) \setminus \{k\} : 1^{(\operatorname{form} x)_i} \right) \cup \{(k; X)\} \in \mathrm{GR} \prod^{\mathsf{Strd}} x \right\} = \left\{ X \in \operatorname{Base} x_k \mid \left(\forall i \in (\operatorname{dom} x) \setminus \{k\} : 1^{(\operatorname{form} x)_i} \not\asymp x_i \right) \wedge X \not\asymp x_k \right\} = \{ X \in \operatorname{Base} x_k \mid X \not\asymp x_k \} = \partial x_k$.

Consequently $\Pr_k^{\mathsf{Strd}} \prod^{\mathsf{Strd}} x = x_k$. □

15.2 Cross-composition product of pointfree funcoids

Zero morphisms of the category of pointfree funcoids are ??.

Proposition 178. Values x_i (for every $i \in \operatorname{dom} x$) can be restored from the value of $\prod^{(C)} x$ provided that x is an indexed family of non-zero pointfree funcoids if $\operatorname{Src} f_i$ (for every $i \in n$) is an atomic lattice and every $\operatorname{Dst} f_i$ has greatest element.

Proof. $\left\langle \prod^{(C)} x \right\rangle \prod^{\mathsf{Strd}} p = \prod^{\mathsf{FCD}}_{i \in n} \langle x_i \rangle p_i$ by the theorem 145.

Since $x_i \neq 0$ there exist p such that $\langle x_i \rangle p_i \neq 0$. Take $k \in n$, $p_i' = p_i$ for $i \neq k$ and $p_k' = q$ for an arbitrary value q; then (using the staroidal projections from the previous subsection)

$$\langle x_k \rangle q = \Pr_k^{\mathsf{Strd}} \prod_{i \in n}^{\mathsf{FCD}} \langle x_i \rangle p_i' = \Pr_k^{\mathsf{Strd}} \left\langle \prod^{(C)} x \right\rangle \prod^{\mathsf{Strd}} p'.$$

So the value of x can be restored from $\prod^{(C)} x$ by this formula. □

15.3 Subatomic product

Proposition 179. Values x_i (for every $i \in \operatorname{dom} x$) can be restored from the value of $\prod^{(A)} x$ provided that x is an indexed family of non-zero funcoids.

Proof. Fix $k \in \operatorname{dom} f$. Let for some filters x and y

$$a = \begin{cases} 1^{\mathfrak{F}(\operatorname{Base}(x))} & \text{if } i \neq k; \\ x & \text{if } i = k \end{cases} \quad \text{and} \quad b = \begin{cases} 1^{\mathfrak{F}(\operatorname{Base}(y))} & \text{if } i \neq k; \\ y & \text{if } i = k. \end{cases}$$

Then $a_k [x_k] b_k \Leftrightarrow \forall i \in \operatorname{dom} f : a_i [x_i] b_i \Leftrightarrow \prod^{\mathsf{RLD}} a \left[\prod^{(A)} x \right] \prod^{\mathsf{RLD}} b$. So we have restored x_k from $\prod^{(A)} x$. □

Conjecture 180. For every funcoid $f : \prod A \rightarrow \prod B$ (where A and B are indexed families of sets) there exists a funcoid $\Pr_k^{(A)} f$ defined by the formula

$$x \left[\Pr_k^{(A)} f \right] y \Leftrightarrow \prod^{\mathsf{RLD}} \left(\begin{cases} 1^{\mathfrak{F}(\operatorname{Base}(x))} & \text{if } i \neq k; \\ x & \text{if } i = k \end{cases} \right) [f] \prod^{\mathsf{RLD}} \left(\begin{cases} 1^{\mathfrak{F}(\operatorname{Base}(y))} & \text{if } i \neq k; \\ y & \text{if } i = k. \end{cases} \right)$$

for:

1. every filters x and y;
2. every principal filters x and y;
3. every atomic filters x and y.