Proof. Using the fact that $\upuparrows f \sqcap^{\mathsf{pStrd}(\mathrm{form}\, f)} \prod^{\mathsf{Strd}} a = \mathrm{StarComp}\left(\upuparrows f; \lambda i \in \mathrm{dom}\, \mathfrak{A}: I_{a_i}^{\mathsf{FCD}(\mathfrak{A}_i)} \right)$ is a completary staroid (theorem 140). □

Theorem 154. $\prod^{\mathsf{Strd}} a \not\asymp^{\mathsf{pStrd}} \prod^{\mathsf{Strd}} b \Leftrightarrow \prod^{\mathsf{Strd}} a \not\asymp^{\mathsf{cStrd}} \prod^{\mathsf{Strd}} b \Leftrightarrow b \in \prod^{\mathsf{Strd}} a \Leftrightarrow a \in \prod^{\mathsf{Strd}} b \Leftrightarrow a \not\asymp b$ for every indexed families a and b of filters on boolean algebras.

Proof. By corollary 66 we have $\prod^{\mathsf{Strd}} b = \upuparrows f$ for some f. Thus as our filtrator is with separable core we can apply the theorem 152 and its corollary. So $\prod^{\mathsf{Strd}} a \not\asymp^{\mathsf{cStrd}} \prod^{\mathsf{Strd}} b \Leftrightarrow a \in \prod^{\mathsf{Strd}} b$ and $\prod^{\mathsf{Strd}} a \not\asymp^{\mathsf{cStrd}} \prod^{\mathsf{Strd}} b \Leftrightarrow a \in \prod^{\mathsf{Strd}} b$. Similarly $\prod^{\mathsf{Strd}} a \not\asymp^{\mathsf{cStrd}} \prod^{\mathsf{Strd}} b \Leftrightarrow b \in \prod^{\mathsf{Strd}} a$. This by the definition of staroidal product is equivalent to $a \not\asymp b$. We are done. □

13 Multireloids

Definition 155. I will call a *multireloid* of the form $A = A_{i \in n}$, where every each A_i is a set, a pair $(f; A)$ where f is a filter on the set $\prod A$.

Definition 156. I will denote $\mathrm{Obj}(f; A) = A$ and $\mathrm{GR}(f; A) = f$ for every multireloid $(f; A)$.

I will denote $\mathsf{RLD}(A)$ the set of multireloids of the form A.

The multireloid $\uparrow^{\mathsf{RLD}(A)} F$ for a binary relation F is defined by the formulas:

$$\mathrm{Obj} \uparrow^{\mathsf{RLD}(A)} F = A \quad \text{and} \quad \mathrm{GR} \uparrow^{\mathsf{RLD}(A)} F = \uparrow^{\prod A} \mathrm{GR}\, F.$$

Let a is a multireloid of the form A and $\mathrm{dom}\, A = n$.

Let every f_i is a reloid with $\mathrm{Src}\, f_i = A_i$.

The star-composition of a with f is a multireloid of the form $\lambda i \in \mathrm{dom}\, A: \mathrm{Src}\, f_i$ defined by the formulas:

$$\mathrm{arity}\, \mathrm{StarComp}(a; f) = n;$$
$$\mathrm{GR}\, \mathrm{StarComp}(a; f) = \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(A)} \mathrm{StarComp}(A; F) \mid \forall A \in a, F \in \prod_{i \in n} f_i \right\};$$
$$\mathrm{Obj}_m\, \mathrm{StarComp}(a; f) = \lambda i \in n: \mathrm{Dst}\, f_i.$$

Theorem 157. Multireloids with above defined compositions form a quasi-invertible category with star-morphisms.

Proof. We need to prove:

1. $\mathrm{StarComp}(\mathrm{StarComp}(m; f); g) = \mathrm{StarComp}(m; \lambda i \in \mathrm{arity}\, m: g_i \circ f_i)$;
2. $\mathrm{StarComp}(m; \lambda i \in \mathrm{arity}\, m: \mathrm{id}_{\mathrm{Obj}_m i}) = m$;
3. $b \not\asymp \mathrm{StarComp}(a; f) \Leftrightarrow a \not\asymp \mathrm{StarComp}(b; f^{\dagger})$

(the rest is obvious).

Really,

1. $\mathrm{StarComp}(\mathrm{StarComp}(A; f); g) = \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(A)} \mathrm{StarComp}(B; G) \mid \forall B \in \mathrm{StarComp}(A; f), G \in \prod_{i \in n} g_i \right\} = \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(A)} \mathrm{StarComp}(\mathrm{StarComp}(A; F); G) \mid \forall A \in a, F \in \prod_{i \in n} f_i, G \in \prod_{i \in n} g_i \right\} = \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(A)} \mathrm{StarComp}(A; G \circ F) \mid \forall A \in a, F \in \prod_{i \in n} f_i, G \in \prod_{i \in n} g_i \right\} = \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(A)} \mathrm{StarComp}(A; H) \mid \forall A \in a, H \in \prod_{i \in n} \lambda i \in n: g_i \circ f_i \right\} = \mathrm{StarComp}(a; \lambda i \in n: g_i \circ f_i)$ (used properties of generalized filter bases) [TODO: More detailed proof.]
2. $\mathrm{StarComp}(m; \lambda i \in \mathrm{arity}\ m: \mathrm{id}_{\mathrm{Obj}_m i}) = \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(A)} \mathrm{StarComp}(A; \mathrm{id}_X) \mid \forall A \in m, X \in \bigcup_{i \in n} \mathscr{P} \mathrm{Obj}_m i \right\} = \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(A)} A \mid \forall A \in a \right\} = m.$
3. Using properties of generalized filter bases,