

- $\text{Src } m = \text{None}$
- $\text{Dst } m = \text{Obj}_m$.
- Compositions of morphisms are defined by the formulas:
 - $g \circ f = \lambda i \in \text{dom } f: g_i \circ f_i$ for our indexed families f and g of morphisms;
 - $f \circ m = \text{StarComp}(m; f)$ for $m \in M$ and a composable indexed family f ;
 - $m \circ \text{id}_{\text{None}} = m$ for $m \in M$;
 - $\text{id}_{\text{None}} \circ \text{id}_{\text{None}} = \text{id}_{\text{None}}$.
- Identity morphisms for an object X are:
 - $\lambda i \in X: \text{id}_{X_i}$ if $X \neq \text{None}$
 - id_{None} if $X = \text{None}$

We need to prove it is really a category.

Proof. We need to prove:

1. Composition is associative
2. Composition with identities complies with the identity law.

Really:

1. $(h \circ g) \circ f = \lambda i \in \text{dom } f: (h_i \circ g_i) \circ f_i = \lambda i \in \text{dom } f: h_i \circ (g_i \circ f_i) = h \circ (g \circ f)$;
 $g \circ (f \circ m) = \text{StarComp}(\text{StarComp}(m; f); g) = \text{StarComp}(m; \lambda i \in \text{arity } m: g_i \circ f_i) =$
 $\text{StarComp}(m; g \circ f) = (g \circ f) \circ m$;
 $f \circ (m \circ \text{id}_{\text{None}}) = f \circ m = (f \circ m) \circ \text{id}_{\text{None}}$.
2. $m \circ \text{id}_{\text{None}} = m$; $\text{id}_{\text{Dst } m} \circ m = \text{StarComp}(m; \lambda i \in \text{arity } m: \text{id}_{\text{Obj}_m i}) = m$. □

Remark 119. I call the above defined category *abrupt category* because (excluding identity morphisms) it allows composition with an $m \in M$ only on the left (not on the right) so that the morphism m is “abrupt” on the right.

By $\llbracket x_0; \dots; x_{n-1} \rrbracket$ I denote an n -tuple.

Definition 120. Pre-category with star morphisms *induced* by a dagger pre-category C is:

- The base category is C .
- Star-morphisms are morphisms of C .
- $\text{arity } f = \{0, 1\}$.
- $\text{Obj}_m = \llbracket \text{Src } m; \text{Dst } m \rrbracket$.
- $\text{StarComp}(m; \llbracket f; g \rrbracket) = g \circ m \circ f^\dagger$.

Let prove it is really a category with star-morphisms.

Proof. We need to prove the associativity law:

$$\text{StarComp}(\text{StarComp}(m; \llbracket f; g \rrbracket); \llbracket p; q \rrbracket) = \text{StarComp}(m; \llbracket p \circ f; q \circ g \rrbracket).$$

Really,

$$\text{StarComp}(g \circ m \circ f^\dagger; \llbracket p; q \rrbracket) = q \circ g \circ m \circ f^\dagger \circ p^\dagger = q \circ g \circ m \circ (p \circ f)^\dagger = \text{StarComp}(m; \llbracket p \circ f; q \circ g \rrbracket). \quad \square$$

Definition 121. Category with star morphisms *induced* by a dagger category C is the above defined pre-category with star-morphisms.

That it is a category (the law of composition with identity) is trivial.