for every $L_0, L_1 \in \prod \text{form} \prod^{(D)} F$ that is $L_0, L_1 \in \prod \text{uncurry}(\text{form} \circ F)$.

Really $L_0 \sqcup L_1 \in \text{GR} \prod^{(D)} F \Leftrightarrow L_0 \sqcup L_1 \in \{\text{uncurry}\, z \mid z \in \prod (\text{GR} \circ F)\}$.

$\exists c \in \{0, 1\}^{\text{arity} \prod^{(D)} F}: (\lambda i \in n: L_{c(i)} i) \in \text{GR} \prod^{(D)} F \Leftrightarrow \exists c \in \{0, 1\}^{\text{arity} \prod^{(D)} F}: \left(\lambda i \in \text{arity} \prod^{(D)} F: L_{c(i)} i\right) \in \{\text{uncurry}\, z \mid z \in \prod (\text{GR} \circ F)\} \Leftrightarrow \exists c \in \{0, 1\}^{\text{arity} \prod^{(D)} F}: \text{curry}\left(\lambda i \in \text{arity} \prod^{(D)} F: L_{c(i)} i\right) \in \prod (\text{GR} \circ F) \Leftrightarrow \exists c \in \{0, 1\}^{\text{arity} \prod^{(D)} F}: \text{curry}\left(\lambda (i; j) \in \text{arity} \prod^{(D)} F: L_{c(i;j)}(i; j)\right) \in \prod (\text{GR} \circ F) \Leftrightarrow \exists c \in \{0, 1\}^{\text{arity} \prod^{(D)} F}: (\lambda i \in \text{dom}\, F: (\lambda j \in \text{dom}\, F_i: L_{c(i;j)}(i; j))) \in \prod (\text{GR} \circ F) \Leftrightarrow \exists c \in \{0, 1\}^{\text{arity} \prod^{(D)} F} \forall i \in \text{dom}\, F: (\lambda j \in \text{dom}\, F_i: L_{c(i;j)}(i; j)) \in \text{GR}\, F_i \Leftrightarrow \forall i \in \text{dom}\, F \exists c \in \{0, 1\}^{\text{dom}\, F_i}: (\lambda j \in \text{dom}\, F_i: L_{c(j)}(i; j)) \in \text{GR}\, F_i \Leftrightarrow \forall i \in \text{dom}\, F \exists c \in \{0, 1\}^{\text{dom}\, F_i}: (\lambda j \in \text{dom}\, F_i: (\text{curry}(L_{c(j)})i)j) \in \text{GR}\, F_i \Leftrightarrow \forall i \in \text{dom}\, F: (\text{curry}(L_0) i \sqcup \text{curry}(L_1) i \in \text{GR}\, F_i) \Leftrightarrow L_0 \sqcup L_1 \in \{\text{uncurry}\, z \mid z \in \prod (\text{GR} \circ F)\}$. □

For staroids it is defined *ordinated product* $\prod^{(\text{ord})}$ as defined in [2].

Obvious 108. If f and g are anchored relations and there exists a bijection φ from arity g to arity f such that $\{F \circ \varphi \mid F \in \text{GR}\, f\} = \text{GR}\, g$, then:

1. f is a pre-staroid iff g is a pre-staroid.
2. f is a staroid iff g is a staroid.
3. f is a completary staroid iff g is a completary staroid.

Corollary 109. Let F is an indexed family of anchored relations and every $(\text{form}\, F)_i$ is a join-semilattice.

1. $\prod^{(\text{ord})} F$ is a pre-staroid if every F_i is a pre-staroid.
2. $\prod^{(\text{ord})} F$ is a staroid if every F_i is a staroid.
3. $\prod^{(\text{ord})} F$ is a completary staroid if every F_i is a completary staroid.

Proof. Use the fact that $\text{GR} \prod^{(\text{ord})} F = \left\{F \circ (\bigoplus (\text{dom} \circ F))^{-1} \mid F \in \text{GR} \prod^{(D)} f\right\}$. □

Definition 110. $f \times^{(\text{ord})} g = \prod^{(\text{ord})} [\![f; g]\!]$.

Remark 111. If f and g are binary funcoids, then $f \times^{(\text{ord})} g$ is ternary.

10 Star categories

Definition 112. A *pre-category with star-morphisms* consists of

1. a pre-category C (*the base pre-category*);
2. a set M (*star-morphisms*);
3. a function “arity” defined on M (how many objects are connected by this multimorphism);
4. a function $\text{Obj}_m: \text{arity}\, m \to \text{Obj}(C)$ defined for every $m \in M$;
5. a function (*star composition*) $(m; f) \mapsto \text{StarComp}(m; f)$ defined for $m \in M$ and f being an $(\text{arity}\, m)$-indexed family of morphisms of C such that $\forall i \in \text{arity}\, m: \text{Src}\, f_i = \text{Obj}_m\, i$ ($\text{Src}\, f_i$ is the source object of the morphism f_i) such that $\text{arity}\, \text{StarComp}(m; f) = \text{arity}\, m$

such that it holds:

1. $\text{StarComp}(m; f) \in M$;
2. (*associativiy law*)

$$\text{StarComp}(\text{StarComp}(m; f); g) = \text{StarComp}(m; \lambda i \in \text{arity}\, m: g_i \circ f_i).$$