Let f is a lower bound for $\left\{ \prod^{\mathsf{Strd}(\mathfrak{F})} A \mid A \in S \right\}$. Thus for every $A \in S$ we have $L \in \mathrm{GR}\, f$ implies $\forall i \in \mathrm{dom}\, \mathfrak{A}: A_i \not\asymp L_i$. Then, by properties of generalized filter bases, $\forall i \in \mathrm{dom}\, \mathfrak{A}: a_i \not\asymp L_i$ that is $L \in \mathrm{GR} \prod^{\mathsf{Strd}(\mathfrak{F})} a$.

So $f \subseteq \prod^{\mathsf{Strd}(\mathfrak{F})} a$. □

Theorem 98. Let $\mathfrak{F}$ is a family of sets of filters on distributive lattices with least elements. Let $a \in \prod \mathfrak{F}$, $S \in \mathscr{P} \prod \mathfrak{F}$ is a generalized filter base, $\bigsqcap S = a$, f is a staroid of the form $\prod \mathfrak{F}$. Then

$$\prod^{\mathsf{Strd}(\mathfrak{F})} a \not\asymp f \Leftrightarrow \forall A \in S: \prod^{\mathsf{Strd}(\mathfrak{A})} A \not\asymp f.$$

Proof. It follows from the previous theorem by properties of generalized filter bases. □

9.1 On products of staroids

Definition 99. $\prod^{(D)} F = \{\mathrm{uncurry}\, z \mid z \in \prod F\}$ (*reindexation product*) for every indexed family F of relations.

Definition 100. *Reindexation product* of an indexed family F of anchored relations is defined by the formulas:

$$\mathrm{form} \prod^{(D)} F = \mathrm{uncurry}(\mathrm{form} \circ F) \quad \text{and} \quad \mathrm{GR} \prod^{(D)} F = \prod (\mathrm{GR} \circ F).$$

Obvious 101.

1. $\mathrm{form} \prod^{(D)} F = \{((i;j); (\mathrm{form}\, F_i)_j) \mid i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i\}$;
2. $\mathrm{GR} \prod^{(D)} F = \{\{((i;j); (zi)j) \mid i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i\} \mid z \in \prod (\mathrm{GR} \circ F)\}$.

Proposition 102. $\prod^{(D)} F$ is an anchored relation if every F_i is an anchored relation.

Proof. We need to prove $\mathrm{GR} \prod{}^{(D)} F \in \mathscr{P} \prod \mathrm{form}\left(\prod{}^{(D)} F\right)$ that is

$\mathrm{GR} \prod{}^{(D)} F \subseteq \prod \mathrm{form}\left(\prod{}^{(D)} F\right)$

$\{\mathrm{uncurry}\, z \mid z \in \prod (\mathrm{GR} \circ F)\} \in \mathscr{P} \prod \{((i;j); (\mathrm{form}\, F_i)_j) \mid i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i\}$;

$\{\mathrm{uncurry}\, z \mid z \in \prod (\mathrm{GR} \circ F)\} \subseteq \prod \{((i;j); (\mathrm{form}\, F_i)_j) \mid i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i\}$

$\{\{((i;j); (zi)j) \mid i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i\} \mid z \in \prod (\mathrm{GR} \circ F)\} \subseteq \prod \{((i;j); (\mathrm{form}\, F_i)_j) \mid i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i\}$;

$\forall z \in \prod (\mathrm{GR} \circ F), i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i: (zi)j \in (\mathrm{form}\, F_i)_j$.

Really, $zi \in \mathrm{GR}\, F_i \subseteq \prod (\mathrm{form}\, F_i)$ and thus $(zi)j \in (\mathrm{form}\, F_i)_j$. □

Remark 103. I suspect that the above proof can be simplified.

Obvious 104. $\mathrm{arity} \prod^{(D)} F = \coprod_{i \in \mathrm{dom}\, F} \mathrm{arity}\, F_i = \{(i;j) \mid i \in \mathrm{dom}\, F, j \in \mathrm{arity}\, F_i\}$.

Definition 105. $f \times^{(D)} g = \prod^{(D)} \llbracket f; g \rrbracket$.

Lemma 106. $\prod^{(D)} F$ is an upper set if every F_i is an upper set.

Proof. We need to prove that $\prod^{(D)} F$ is an upper set. Let $a \in \prod^{(D)} F$ and an anchored relation $b \sqsupseteq a$ of the same form as a. We have $a = \mathrm{uncurry}\, z$ for some $z \in \prod F$ that is $a(i;j) = (zi)j$ for all $i \in \mathrm{dom}\, F$ and $j \in \mathrm{dom}\, F_i$ where $zi \in F_i$. Also $b(i;j) \sqsupseteq a(i;j)$. Thus $(\mathrm{curry}\, b)i \sqsupseteq zi$; $\mathrm{curry}\, b \in \prod F$ because every F_i is an upper set and so $b \in \prod^{(D)} F$. □

Proposition 107. Let F is an indexed family of anchored relations and every $(\mathrm{form}\, F)_i$ is a join-semilattice.

1. $\prod^{(D)} F$ is a pre-staroid if every F_i is a pre-staroid.