is defined as a staroid

$$q\in B=\mathsf{pStrd}(\lambda i\in\operatorname{dom} f\colon \mathsf{RLD}(\operatorname{Src} f_i;\operatorname{Src} g_i))$$

such that

$$q=\upuparrows^{(B;C;\uparrow^B)}\downdownarrows^{(A;C;\uparrow^A)}p$$

where $C=\mathsf{pStrd}\left(\prod_{i\in\operatorname{dom} f}\operatorname{Src} f_i;\ \prod_{i\in\operatorname{dom} f}\operatorname{Dst} f_i\right)$.

Definition 68. We will define *displaced product* of a family f of funcoids by the formula: $\prod^{(\mathrm{DP})} f=\mathrm{DP}\left(\prod^{(C)} f\right)$.

Remark 69. The interesting aspect of displaced product of funcoids is that displaced product of pointfree funcoids is a funcoid (not just a pointfree funcoid).

7 Multifuncoids

Definition 70. I call an *pre-multifuncoid sketch* f of the form $\mathfrak{A}$ (where every $\mathfrak{A}_i$ is a poset) the pair $(\mathfrak{A};\alpha)$ where for every $i\in\operatorname{dom}\alpha$

$$\alpha_i\colon \prod \mathfrak{A}|_{(\operatorname{dom}\mathfrak{A})\setminus\{i\}}\rightarrow\mathfrak{A}_i.$$

I denote $\langle f\rangle=\alpha$.

Definition 71. A pre-multifuncoid sketch *on powersets* is a pre-multifuncoid sketch such that every $\mathfrak{A}_i$ is the set of filters on a powerset.

Definition 72. I will call a *pre-multifuncoid* a pre-multifuncoid sketch such that for every i, $j\in\operatorname{dom}\mathfrak{A}$ and $L\in\prod\mathfrak{A}$

$$L_i\not\asymp\alpha_i\,L|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L_j\not\asymp\alpha_j\,L|_{(\operatorname{dom} L)\setminus\{j\}}. \tag{4}$$

Definition 73. Let $\mathfrak{A}$ is an indexed family of starrish posets. The pre-staroid *corresponding* to a pre-multifuncoid f is $[f]$ defined by the formula:

$$\operatorname{form}[f]=\mathfrak{A}\quad\text{and}\quad L\in\operatorname{GR}[f]\Leftrightarrow L_i\not\asymp\langle f\rangle_i\,L|_{(\operatorname{dom} L)\setminus\{i\}}.$$

Proposition 74. The pre-staroid corresponding to a pre-multifuncoid is really a pre-staroid.

Proof. By the definition of starrish posets. □

Definition 75. I will call a *multifuncoid* a pre-multifuncoid to which corresponds a staroid.

Definition 76. I will call a *completary multifuncoid* a pre-multifuncoid to which corresponds a completary staroid.

Theorem 77. Fix some indexed family $\mathfrak{A}$ of boolean lattices. The the set of multifuncoids g bijectively corresponds to set of pre-staroids f of form $\mathfrak{A}$ by the formulas:

1. $f=[g]$ for every $i\in\operatorname{dom}\mathfrak{A}$, $L\in\prod\mathfrak{A}$;
2. $\partial\langle g\rangle_i\,L=(\operatorname{val} f)_iL$.

Proof. Let f is a pre-staroid of the form $\mathfrak{A}$. If α is defined by the formula $\alpha_i\,L=\langle f\rangle_iL$ then $\partial\alpha_iL=(\operatorname{val} f)_iL$. Then

$$L_i\not\asymp\alpha_i\,L|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L\in f\Leftrightarrow L_j\not\asymp\alpha_j\,L|_{(\operatorname{dom} L)\setminus\{j\}}.$$

For the staroid f' defined by the formula $L\in f'\Leftrightarrow L_i\not\asymp\alpha_i\,L|_{(\operatorname{dom} L)\setminus\{i\}}$ we have:

$$L\in f'\Leftrightarrow L_i\in\partial\alpha_i\,L|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L_i\in(\operatorname{val} f)_iL|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L\in f;$$