3. completary staroids.

Proof. f is a finitary pre-staroid $\Rightarrow$ f is a finitary completary staroid.

f is a finitary completary staroid $\Rightarrow$ f is a finitary staroid.

f is a finitary staroid $\Rightarrow$ f is a finitary pre-staroid. □

Definition 59. We will denote the set of staroids, pre-staroids, and completary staroids of a form $\mathfrak{A}$ correspondingly as $\mathsf{Strd}(\mathfrak{A})$, $\mathsf{pStrd}(\mathfrak{A})$, and $\mathsf{cStrd}(\mathfrak{A})$.

6 Upgrading and downgrading a set regarding a filtrator

Let fix a filtrator $(\mathfrak{A}; \mathfrak{Z})$.

Definition 60. $\downdownarrows f = f \cap \mathfrak{Z}$ for every $f \in \mathscr{P}\mathfrak{A}$ (*downgrading* f).

Definition 61. $\upuparrows f = \{ L \in \mathfrak{A} \mid \operatorname{up} L \subseteq f \}$ for every $f \in \mathscr{P}\mathfrak{Z}$ (*upgrading* f).

Obvious 62. $a \in \upuparrows f \Leftrightarrow \operatorname{up} a \subseteq f$ for every $f \in \mathscr{P}\mathfrak{Z}$ and $a \in \mathfrak{A}$.

Proposition 63. $\downdownarrows \upuparrows f = f$ if f is an upper set.

Proof. $\downdownarrows \upuparrows f = \upuparrows f \cap \mathfrak{Z} = \{ L \in \mathfrak{Z} \mid \operatorname{up} L \subseteq f \} = \{ L \in \mathfrak{Z} \mid \operatorname{up} L \in f \} = f \cap \mathscr{P}\mathfrak{Z} = f$. □

6.1 Upgrading and downgrading staroids

Let fix a family $(\mathfrak{A}; \mathfrak{Z})$ of filtrators.

For a graph f of a staroid define $\downdownarrows f$ and $\upuparrows f$ taking the filtrator of $(\prod \mathfrak{A}; \prod \mathfrak{Z})$.

For a staroid f define:

$$\operatorname{form} \downdownarrows f = \mathfrak{Z} \quad \text{and} \quad \operatorname{GR} \downdownarrows f = \downdownarrows \operatorname{GR} f;$$
$$\operatorname{form} \upuparrows f = \mathfrak{A} \quad \text{and} \quad \operatorname{GR} \upuparrows f = \upuparrows \operatorname{GR} f.$$

Proposition 64. $(\operatorname{val} \downdownarrows f))_i L = (\operatorname{val} f)_i L \cap \mathfrak{Z}_i$ for every $L \in \prod \mathfrak{Z}|_{(\operatorname{arity} f) \setminus \{i\}}$.

Proof. $(\operatorname{val} \downdownarrows f))_i L = \{ X \in (\operatorname{form} f)_i \mid L \cup \{(i; X)\} \in \operatorname{GR} f \cap \prod \mathfrak{Z} \} = \{ X \in \mathfrak{Z}_i \mid L \cup \{(i; X)\} \in \operatorname{GR} f \} = (\operatorname{val} f)_i L \cap \mathfrak{Z}_i$. □

Proposition 65. Let $(\mathfrak{A}_i; \mathfrak{Z}_i)$ are finitely join-closed filtrators with both the base and the core being join-semilattices. If f is a staroid of the form $\mathfrak{A}$, then $\downdownarrows f$ is a staroid of the form $\mathfrak{Z}$.

Proof. Let f is a a staroid.

We need to prove that $(\operatorname{val} \downdownarrows f)_i L$ is a free star. It follows from the last proposition and the fact that it is join-closed. □

Proposition 66. $\prod^{\mathsf{Strd}} a = \upuparrows \downdownarrows \prod^{\mathsf{Strd}} a$ if each $a_i \in \mathfrak{A}_i$ (for $i \in n$ where n is some index set) where $\mathfrak{A}_i$ is a separable poset with least element.

Proof. $\upuparrows \downdownarrows \prod^{\mathsf{Strd}} a = \left\{ L \in \prod \mathfrak{A} \mid L \subseteq \prod^{\mathsf{Strd}} a \right\} = \{ L \in \prod \mathfrak{A} \mid \forall K \in L : K \not\asymp a \} = \{ L \in \prod \mathfrak{A} \mid L \not\asymp a \} = \prod^{\mathsf{Strd}} a$ (taken into account that $\prod \mathfrak{A}$ is a separable poset). □

6.2 Displacement

Definition 67. Let f is an indexed family of pointfree funcoids. The *displacement* of the pre-staroid

$$p \in A = \mathsf{pStrd}(\lambda i \in \operatorname{dom} f : \mathsf{FCD}(\operatorname{Src} f_i; \operatorname{Src} g_i))$$