Proof. We need to prove that $\lambda i\in\operatorname{dom}\mathfrak{A}: a_i\#b_i=\bigsqcap\{z\in\prod\mathfrak{A}\mid z\sqsubseteq a\wedge z\asymp b\}$.

To prove it is enough to show $a_i\#b_i=\bigsqcap\{z_i\mid z\in\prod\mathfrak{A}, z\sqsubseteq a\wedge z\asymp b\}$ that is $a_i\#b_i=\bigsqcap\{z\in\mathfrak{A}_i\mid z\sqsubseteq a_i\wedge\forall j\in\operatorname{dom}\mathfrak{A}: z_j\asymp b_j\}$ that is $a_i\#b_i=\bigsqcap\{z\in\mathfrak{A}_i\mid z\sqsubseteq a_i\wedge z\asymp b_i\}$ (take $z_i=0$ for $j\neq i$) what is true by definition. □

Proposition 40. Let every $\mathfrak{A}_i$ is a poset with least element and a_i^* is defined. Then $a^*=\lambda i\in n: a_i^*$.

Proof. We need to prove that $\lambda i\in\operatorname{dom}\mathfrak{A}: a_i^*=\bigsqcup\{c\in\mathfrak{A}\mid c\asymp a\}$. To prove this it is enough to show that $a_i^*=\bigsqcup\{c_i\mid c\in\mathfrak{A}, c\asymp a\}$ that is $a_i^*=\bigsqcup\{c_i\mid c\in\mathfrak{A},\forall j\in n: c_j\asymp a_j\}$ that is $a_i^*=\bigsqcup\{c_i\mid c\in\mathfrak{A}, c_i\asymp a_i\}$ (take $c_i=0$ for $j\neq i$) that is $a_i^*=\bigsqcup\{c\in\mathfrak{A}\mid c\asymp a_i\}$ what is true by definition. □

Corollary 41. Let every $\mathfrak{A}_i$ is a poset with least element and a_i^+ is defined. Then $a^+=\lambda i\in n: a_i^+$.

Proof. By duality. □

5 Definition of staroids

Let n be a set. As an example, n may be an ordinal, n may be a natural number, considered as a set by the formula $n=\{0,\ldots,n-1\}$. Let $\mathfrak{A}=\mathfrak{A}_{i\in n}$ is a family of posets indexed by the set n.

Definition 42. I will call an *anchored relation* a pair $f=(\operatorname{form} f;\operatorname{GR} f)$ of a family $\operatorname{form}(f)$ of sets indexed by the some index set and a relation $\operatorname{GR}(f)\in\mathscr{P}\prod\operatorname{form}(f)$. I call $\operatorname{GR}(f)$ the *graph* of the anchored relation f. I denote $\operatorname{Anch}(\mathfrak{A})$ the set of small anchored relations of the form $\mathfrak{A}$.

Definition 43. An anchored relation *on powersets* is an anchored relation f such that every $(\operatorname{form} f)_i$ is a powerset.

I will denote $\operatorname{arity} f=\operatorname{dom}\operatorname{form} f$.

Definition 44. Every set of anchored relations of the same form constitutes a poset by the formula $f\sqsubseteq g\Leftrightarrow\operatorname{GR} f\subseteq\operatorname{GR} g$.

Definition 45. An anchored relation is an *anchored relation between posets* when every $(\operatorname{form} f)_i$ is a poset.

Definition 46. Let f is an anchored relation. For every $i\in\operatorname{arity} f$ and $L\in\prod((\operatorname{form} f)|_{(\operatorname{arity} f)\setminus\{i\}})$

$$(\operatorname{val} f)_i L=\{X\in(\operatorname{form} f)_i\mid L\cup\{(i;X)\}\in\operatorname{GR} f\}$$

("val" is an abbreviation of the word "value".)

Obvious 47. $X\in(\operatorname{val} f)_i L\Leftrightarrow L\cup\{(i;X)\}\in\operatorname{GR} f$.

Proposition 48. f can be restored knowing $\operatorname{form}(f)$ and $(\operatorname{val} f)_i$ for some $i\in n$.

Proof. $\operatorname{GR} f=\{K\in\prod\operatorname{form} f\mid K\in\operatorname{GR} f\}=\{L\cup\{(i;X)\}\mid L\in\prod(\operatorname{form} f)|_{(\operatorname{arity} f)\setminus\{i\}}, X\in(\operatorname{form} f)_i, L\cup\{(i;X)\}\in\operatorname{GR} f\}=\{L\cup\{(i;X)\}\mid L\in\prod(\operatorname{form} f)|_{(\operatorname{arity} f)\setminus\{i\}}, X\in(\operatorname{val} f)_i L\}$. □

Definition 49. A *pre-staroid* is an anchored relation f between poset such that $(\operatorname{val} f)_i L$ is a free star for every $i\in\operatorname{arity} f$, $L\in\prod(\operatorname{form} f)|_{(\operatorname{arity} f)\setminus\{i\}}$.

Definition 50. A *staroid* is a pre-staroid whose graph is an upper set (on the poset if anchored relations of the form of this pre-staroid).

Proposition 51. If $L\in\prod\operatorname{form} f$ and $L_i=0^{(\operatorname{form} f)_i}$ for some $i\in\operatorname{arity} f$ then $L\notin f$ if f is an pre-staroid.

Proof. Let $K=L|_{(\operatorname{arity} f)\setminus\{i\}}$. We have $0\notin(\operatorname{val} f)_i K$; $K\cup\{(i;0)\}\notin f$; $L\notin f$. □