$\operatorname{im}(\nu \circ f|_{\langle\mu\rangle\{x\}}) = \langle\nu\rangle\langle f\rangle\langle\mu\rangle\{x\}$;

$$\begin{aligned} \nu \circ f|_{\langle\mu\rangle\{x\}} &\subseteq \\ \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \operatorname{im}(\nu \circ f|_{\langle\mu\rangle\{x\}}) &= \\ \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\langle f\rangle\langle\mu\rangle\{x\} &\subseteq \\ \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\langle f\rangle\{x\}. \end{aligned}$$

So $\nu \circ f|_{\langle\mu\rangle\{x\}} = \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\langle f\rangle\{x\}$.

Thus $\operatorname{xlim}_x f = \{(\langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\langle f\rangle\{x\}) \circ r \mid r \in G\} = \tau(fx)$. □

Remark 22. Without the requirement of $\langle\mu\rangle\{x\} \supseteq \{x\}$ the last theorem would not work in the case of removable singularity.

Theorem 23. *Let $\nu \subseteq \nu \circ \nu$. If $f|_{\langle\mu\rangle\{x\}} \xrightarrow{\nu} \{y\}$ then $\operatorname{xlim}_x f = \tau(y)$.*

Proof. $\operatorname{im} f|_{\langle\mu\rangle\{x\}} \subseteq \langle\nu\rangle\{y\}$; $\langle f\rangle\langle\mu\rangle\{x\} \subseteq \langle\nu\rangle\{y\}$;

$$\begin{aligned} \nu \circ f|_{\langle\mu\rangle\{x\}} &\supseteq \\ (\langle\nu\rangle\{y\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\}) \circ f|_{\langle\mu\rangle\{x\}} &= \\ \langle (f|_{\langle\mu\rangle\{x\}})^{-1}\rangle\langle\nu\rangle\{y\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\} &= \\ \langle I_{\langle\mu\rangle\{x\}} \circ f^{-1}\rangle\langle\nu\rangle\{y\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\} &\supseteq \\ \langle I_{\langle\mu\rangle\{x\}} \circ f^{-1}\rangle\langle f\rangle\langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\} &= \\ \langle I_{\langle\mu\rangle\{x\}}\rangle\langle f^{-1} \circ f\rangle\langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\} &\supseteq \\ \langle I_{\langle\mu\rangle\{x\}}\rangle\langle I_{\langle\mu\rangle\{x\}}\rangle\langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\} &= \\ \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\}. \end{aligned}$$

On the other hand, $f|_{\langle\mu\rangle\{x\}} \subseteq \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\}$;

$\nu \circ f|_{\langle\mu\rangle\{x\}} \subseteq \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\langle\nu\rangle\{y\} \subseteq \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\}$.

So $\nu \circ f|_{\langle\mu\rangle\{x\}} = \langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\}$.

$\operatorname{xlim}_x f = \{\nu \circ f|_{\langle\mu\rangle\{x\}} \circ r \mid r \in G\} \supseteq \{(\langle\mu\rangle\{x\} \times^{\mathsf{FCD}} \langle\nu\rangle\{y\}) \circ r \mid r \in G\} = \tau(y)$. □

Corollary 24. *If $\lim^{\nu}_{\langle\mu\rangle\{x\}} f = y$ then $\operatorname{xlim}_x f = \tau(y)$.*

We have bijective τ if $\langle\nu\rangle\{y_1\} \cap^{\mathfrak{F}} \langle\nu\rangle\{y_2\} \neq \emptyset$ that is if ν is T_1-separable.

5.2 Generalized limits as a generalization of limits

When τ is bijective, using the procedure described in appendix B in [1] we can equate points of the space with certain generalized limits.

Thus we can use only "lim" to denote all kinds of limits and eliminate "xlim" notation.

5.3 Yet to do

We need to study generalized limits of composition of functions. It is yet unclear how to define this.

We should introduce n-ary functions extended on values which generalized limits take, so that we could be able for example to add two generalized limits.

We should study differential equations generalized for the derivative of non-smooth functions.

Need examples with metric spaces.

Bibliography

[1] Victor Porton. Filters on posets and generalizations. At http://www.mathematics21.org/binaries/filters.pdf.

[2] Victor Porton. Funcoids and reloids. At http://www.mathematics21.org/binaries/funcoids-reloids.pdf.