Definition 3. *A funcoid f converges to a filter object $\mathcal{A}$ on a filter object $\mathcal{B}$ regarding a funcoid μ iff $f|_{\mathcal{B}}\xrightarrow{\mu}\mathcal{A}$.*

Remark 4. We can define also convergence for a reloid f: $f\xrightarrow{\mu}\mathcal{A}\Leftrightarrow \operatorname{im} f\subseteq\langle\mu\rangle\mathcal{A}$ or what is the same $f\xrightarrow{\mu}\mathcal{A}\Leftrightarrow(\mathsf{FCD})f\xrightarrow{\mu}\mathcal{A}$.

Theorem 5. *Let f, g, μ, ν are funcoids, $\mathcal{A}$ is a filter object. If $f\xrightarrow{\mu}\mathcal{A}$,*

$$g|_{\langle\mu\rangle\mathcal{A}}\in\mathrm{C}(\mu\cap(\langle\mu\rangle\mathcal{A})^2;\nu)$$

and $\langle\mu\rangle\mathcal{A}\supseteq\mathcal{A}$ then $g\circ f\xrightarrow{\nu}\langle g\rangle\mathcal{A}$.

Proof. $\operatorname{im} f\subseteq\langle\mu\rangle\mathcal{A}$; $\langle g\rangle\operatorname{im} f\subseteq\langle g\rangle\langle\mu\rangle\mathcal{A}$; $\operatorname{im}(g\circ f)\subseteq\langle g|_{\langle\mu\rangle\mathcal{A}}\rangle\langle\mu\rangle\mathcal{A}$; $\operatorname{im}(g\circ f)\subseteq\langle g|_{\langle\mu\rangle\mathcal{A}}\rangle\langle\mu\cap(\langle\mu\rangle\mathcal{A})^2\rangle\mathcal{A}$; $\operatorname{im}(g\circ f)\subseteq\langle g|_{\langle\mu\rangle\mathcal{A}}\circ(\mu\cap(\langle\mu\rangle\mathcal{A})^2)\rangle\mathcal{A}$; $\operatorname{im}(g\circ f)\subseteq\langle\nu\circ g|_{\langle\mu\rangle\mathcal{A}}\rangle\mathcal{A}$; $\operatorname{im}(g\circ f)\subseteq\langle\nu\circ g\rangle\mathcal{A}$; $\operatorname{im}(g\circ f)\subseteq\langle\nu\rangle\langle g\rangle\mathcal{A}$; $g\circ f\xrightarrow{\nu}\langle g\rangle\mathcal{A}$. □

Corollary 6. *Let f, g, μ, ν are funcoids, $\mathcal{A}$ is a filter object. If $f\xrightarrow{\mu}\mathcal{A}$, $g\in\mathrm{C}(\mu;\nu)$ and $\langle\mu\rangle\mathcal{A}\supseteq\mathcal{A}$ then $g\circ f\xrightarrow{\nu}\langle g\rangle\mathcal{A}$.*

Proof. From the last theorem and a theorem in [3]. □

The following is the theorem about convergence of a continuous funcoid:

Theorem 7. *If $f\in\mathrm{C}(\mu;\nu)$ then $f|_{\langle\mu\rangle\mathcal{A}}\xrightarrow{\nu}\langle f\rangle\mathcal{A}$ (for any funcoids μ and ν and a filter object $\mathcal{A}$).*

Proof. $f|_{\langle\mu\rangle\mathcal{A}}\xrightarrow{\nu}\langle f\rangle\mathcal{A}\Leftrightarrow\operatorname{im} f|_{\langle\mu\rangle\mathcal{A}}\subseteq\langle\nu\rangle\langle f\rangle\mathcal{A}\Leftrightarrow\langle f\rangle\langle\mu\rangle\mathcal{A}\subseteq\langle\nu\rangle\langle f\rangle\mathcal{A}\Leftrightarrow\langle f\circ\mu\rangle\mathcal{A}\subseteq\langle\nu\circ f\rangle\mathcal{A}\Leftarrow f\circ\mu\subseteq\nu\circ f\Leftarrow f\in\mathrm{C}(\mu;\nu)$. □

4 Limit

Definition 8. $\lim^\mu f=a$ *iff $f\xrightarrow{\mu}\{a\}$ for a T_2-separable funcoid μ and a non-empty funcoid f.*

It is defined correctly, that is f has no more than one limit.

Proof. Let $\lim^\mu f=a$ and $\lim^\mu f=b$. Then $\operatorname{im} f\subseteq\langle\mu\rangle\{a\}$ and $\operatorname{im} f\subseteq\langle\mu\rangle\{b\}$.

Because $f\neq 0$ we have $\operatorname{im} f\neq 0$; $\langle\mu\rangle\{a\}\cap\langle\mu\rangle\{b\}\neq 0$; $\{b\}\cap\langle\mu^{-1}\rangle\langle\mu\rangle\{a\}\neq 0$; $\{b\}\cap\langle\mu^{-1}\circ\mu\rangle\{a\}\neq 0$; $\{a\}\,[\mu^{-1}\circ\mu]\,\{b\}$. Because μ is T_2-separable we have $a=b$. □

Definition 9. $\lim^\mu_{\mathcal{B}} f=\lim^\mu(f|_{\mathcal{B}})$.

Remark 10. We can also in an obvious way define limit of a reloid.

5 Generalized limit

5.1 The definition

Let μ and ν are funcoids [2], G is a group of functions.

Let D is a set such that $\forall r\in G\colon\operatorname{im} r\subseteq D\wedge\forall x,y\in D\exists r\in G\colon r(x)=y$.

We require that μ and any $r\in G$ commute, that is $\mu\circ r=r\circ\mu$.

We require for every $y\in\mho$

$$\nu\supseteq\langle\nu\rangle\{y\}\times^{\mathsf{FCD}}\langle\nu\rangle\{y\}\tag{1}$$

Remark 11. The formula (1) usually works if ν is a proximity. It does not work if μ is a pretopology or preclosure.