

If a formula  $F(x_0, \dots, x_n)$  holds for every poset  $\mathfrak{A}$  then it also holds for product order  $\prod \mathfrak{A}$ . (What about infinite formulas like complete lattice joins and meets?) Moreover  $F(x_0, \dots, x_n) = \lambda i \in n : F(x_0, i, \dots, x_n, i)$  (confused logical forms and functions). It looks like a promising approach, but how to define it exactly? For example,  $F$  may be a form always true for boolean lattices or for Heyting lattices, or whatsoever. How one theorem can encompass all kinds of lattices and posets? We may attempt to restrict to (partial) functions determined by order. (This is not enough, because we can define an operation restricting  $\setminus$  defined only for posets of cardinality above or below some cardinal  $\kappa$ . For such restricted  $\setminus$  the above formula does not work.) See also <https://portonmath.wordpress.com/2016/01/12/a-conjecture-about-product-order-and-logic/>. It seems that TODD TRIMBLE shows a general category-theoretic way to describe this: <https://nforum.ncatlab.org/discussion/6887/operations-on-product-order/>.

Get results from <http://ncatlab.org/toddtrimble/published/topogeny>.

What about distributivity of quasicomplements over meets and joins for the filtrator of functors? Seems like nontrivial conjectures.

Conjecture: Each filtered filtrator is isomorphic to a primary filtrator. (If it holds, then primary and filtered filtrators are the same!)

Add analog of the last item of the theorem about co-complete functors for point-free functors.

Generalize theorems about  $\text{RLD}(A; B)$  as  $\mathcal{F}(A \times B)$  in order to clean up the notation (for example in the chapter “Functors are filters”).

Define reloids as a filtrator whose core is an ordered semigroup. This way reloids can be described in several isomorphic ways (just like primary filtrators are both filtrators of filters, of ideals, etc.) Is it enough to describe all properties of reloids? Well, it is not a semigroup, it is a precategory. It seems that we also need functions  $\text{dom}$  and  $\text{im}$  into partially ordered sets and “reversion” (dagger).

<http://mathoverflow.net/a/191381/4086> says that  $n$ -staroids can be identified with certain ideals!

To relax theorem conditions and definition, we can define *protofunctors* as arbitrary pairs  $(\alpha; \beta)$  of functions between two posets. For protofunctors composition and reverse are defined.

Add examples of functors to demonstrate their power:  $D \sqcup T$  ( $D$  is a digraph  $T$  is a topological space),  $T \sqcap \left\{ \begin{smallmatrix} (x;y) \\ y \geq x \end{smallmatrix} \right\}$  as “one-side topology” and also a circle made from its  $\pi$ -length segment.

Say explicitly that pseudodifference is a special case of difference.

For pointfree functors, if  $f : \mathfrak{A} \rightarrow \mathfrak{B}$  exists, then existence of least element of  $\mathfrak{A}$  is equivalent to existence of least element of  $\mathfrak{B}$ :  $y \neq \langle f \rangle \perp^{\mathfrak{A}} \Leftrightarrow \perp^{\mathfrak{A}} \neq \langle f^{-1} \rangle y \Leftrightarrow 0$ . Thus  $\langle f \rangle y \simeq \langle f \rangle y$  and so  $\langle f \rangle y = \perp^{\mathfrak{B}}$ . Can a similar statement be made that  $\mathfrak{A}$  being join-semilattice implies  $\mathfrak{B}$  being join-semilattice (at least for separable posets)? If yes, this could allow to shorten some theorem conditions. It seems we can produce a counter-example for non-separable posets by replacing an element with another element with the same full star.

Develop Todd Trimble’s idea to represent functors as a relation  $\xi$  further: Define functor as a function from sets to sets of sets  $\xi(A \sqcup B) = \xi A \cap \xi B$  and  $\xi \perp = \emptyset$ .

Denote the set of least elements as  $\text{Least}$ . (It is either an one-element set or empty set.)

Show that cross-composition product is a special case of infimum product.