# Generalization in ZF*

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**Abstract**

In the framework of ZF are formally considered generalizations, such as whole numbers generalizing natural numbers, rational numbers generalizing whole numbers, real numbers generalizing rational numbers, complex numbers generalizing real numbers, etc. The formal consideration of this may be especially useful for computer proof assistants. The article is accompanied with usable Isabelle/ZF code.

**Keywords:** ZF, ZFC, generalization, formalistics, bijection, injection

**A.M.S. subject classification:** 03B30, 03E20

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## 1 Preface

In this article I define the notion of generalization in ZF set theory.

Examples: whole numbers generalizing natural numbers, rational numbers generalizing whole numbers, real numbers generalizing rational numbers, complex numbers generalizing real numbers, etc.

I also have implemented this theory in Isabelle/ZF proof assistant (see http://isabelle.in.tum.de/index.html) formal language. This implementation is a candidate to be actually useful in development of Isabelle/ZF theories. However my implementation may be not ideal and may need further polishing before actual using in practice.

## 2 Current state of the issue

Whilst in informal mathematics is actively used the notion of generalization, it usually refers to intuition of the reader rather than to a formal consideration. This article provides a formal consideration compatible with usual intuitive notion of generalization.

In Isabelle proof assistant (both Isabelle/ZF and Isabelle/HOL) currently different sets are defined independently. For example in Isabelle/ZF there are sets `nat` (natural numbers) and `int` (integer numbers). It is not assumed that `nat` is a subset of `int`. The only connection between these provided is the function `inf_of` which transforms a natural number into the corresponding integer. Also as an effect of this, operations (such as additions) are defined differently and independently for naturals and integers.

∗. This document has been written using the GNU TeXmacs text editor (see `www.texmacs.org`).