Proof It's enough to prove $\mathcal{A} \ltimes \mathcal{B} \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$.

Let $\Delta_+ = \Delta \cap \uparrow^{\mathbb{R}} (0; +\infty)$. Let $\mathcal{A} = \mathcal{B} = \Delta_+$.

Let $K = (\leqslant)|_{\mathbb{R}\times\mathbb{R}}$.

Obviously $K \notin \operatorname{up}\left(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}\right)$.

$\mathcal{A} \ltimes \mathcal{B} \subseteq \uparrow^{\mathsf{RLD}(\operatorname{Base}(\mathcal{A});\operatorname{Base}(\mathcal{B}))} K$ and thus $K \in \operatorname{up}(\mathcal{A} \ltimes \mathcal{B})$ because $\uparrow^{\mathsf{FCD}(\operatorname{Base}(\mathcal{A});\operatorname{Base}(\mathcal{B}))} K \supseteq \Delta_+ \times^{\mathsf{FCD}} \uparrow^{\operatorname{Base}(\mathcal{B})} B = \mathcal{A} \times^{\mathsf{FCD}} \uparrow^{\operatorname{Base}(\mathcal{B})} B$ for $B = (0; +\infty)$.

Thus $\mathcal{A} \ltimes \mathcal{B} \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$. □

Example 16 $\mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B} \subset \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ for some f.o. $\mathcal{A}$, $\mathcal{B}$.

Proof This follows from the above example. □

Proposition 53 *$(\mathcal{A} \ltimes \mathcal{B}) \cap (\mathcal{A} \rtimes \mathcal{B}) = \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B}$ for every f.o. $\mathcal{A}$, $\mathcal{B}$.*

Proof $(\mathcal{A} \ltimes \mathcal{B}) \cap (\mathcal{A} \rtimes \mathcal{B}) \subseteq \bigcap\{\uparrow^{\mathsf{RLD}(\operatorname{Base}(\mathcal{A});\operatorname{Base}(\mathcal{B}))} f \mid f \in \mathscr{P}(\operatorname{Base}(\mathcal{A}) \times \operatorname{Base}(\mathcal{B})), \uparrow^{\mathsf{FCD}(\operatorname{Base}(\mathcal{A});\operatorname{Base}(\mathcal{B}))} f \supseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\} = \bigcap\left\{\uparrow^{\mathsf{RLD}(\operatorname{Base}(\mathcal{A});\operatorname{Base}(\mathcal{B}))} f \mid f \in \operatorname{up}\left(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\right)\right\} = (\mathsf{RLD})_{\mathrm{out}}\left(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\right) = \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B}$.

To finish the proof we need to show $\mathcal{A} \ltimes \mathcal{B} \supseteq \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B}$ and $\mathcal{A} \rtimes \mathcal{B} \supseteq \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B}$. By symmetry it's enough to show $\mathcal{A} \ltimes \mathcal{B} \supseteq \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B}$ what is proved above. □

Example 17 $(\mathcal{A} \ltimes \mathcal{B}) \cup (\mathcal{A} \rtimes \mathcal{B}) \subset \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ for some f.o. $\mathcal{A}$, $\mathcal{B}$.

Proof (based on [3]) Let $\mathcal{A} = \mathcal{B} = \Omega(\mathbb{N})$. It's enough to prove $(\mathcal{A} \ltimes \mathcal{B}) \cup (\mathcal{A} \rtimes \mathcal{B}) \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$.

Let $X \in \operatorname{up} \mathcal{A}$, $Y \in \operatorname{up} \mathcal{B}$ that is $X \in \Omega(\mathbb{N})$, $Y \in \Omega(\mathbb{N})$.

Removing one element x from X produces a set P. Removing one element y from Y produces a set Q. Obviously $P \in \Omega(\mathbb{N})$, $Q \in \Omega(\mathbb{N})$.

Obviously $(P \times \mathbb{N}) \cup (\mathbb{N} \times Q) \in \operatorname{up}((\mathcal{A} \ltimes \mathcal{B}) \cup (\mathcal{A} \rtimes \mathcal{B}))$.

$(P \times \mathbb{N}) \cup (\mathbb{N} \times Q) \not\supseteq X \times Y$ because $(x; y) \in X \times Y$ but $(x; y) \notin (P \times \mathbb{N}) \cup (\mathbb{N} \times Q)$.

Thus $(P \times \mathbb{N}) \cup (\mathbb{N} \times Q) \notin \operatorname{up}\left(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}\right)$ by properties of filter bases. □

Example 18 $(\mathsf{RLD})_{\mathrm{out}}(\mathsf{FCD})f \neq f$ for some convex reloid f.

Proof Let $f = \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ where $\mathcal{A}$ and $\mathcal{B}$ are from the previous example.

$(\mathsf{FCD})(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) = \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ by the proposition 33.

So $(\mathsf{RLD})_{\mathrm{out}}(\mathsf{FCD})(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) = (\mathsf{RLD})_{\mathrm{out}}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) = \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B} \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$. □