Example 12 There exist funcoids f and g such that

$$(\mathsf{RLD})_{\mathrm{out}}(g \circ f) \neq (\mathsf{RLD})_{\mathrm{out}} g \circ (\mathsf{RLD})_{\mathrm{out}} f.$$

Proof Take $f = I^{\mathsf{FCD}}_{\Omega(\mathbb{N})}$ and $g = 1^{\mathfrak{F}(\mathbb{N})} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\alpha\}$ for some $\alpha \in \mathbb{N}$. Then $(\mathsf{RLD})_{\mathrm{out}} f = 0^{\mathsf{RLD}(\mathbb{N};\mathbb{N})}$ and thus $(\mathsf{RLD})_{\mathrm{out}} g \circ (\mathsf{RLD})_{\mathrm{out}} f = 0^{\mathsf{RLD}(\mathbb{N};\mathbb{N})}$.

We have $g \circ f = \Omega(\mathbb{N}) \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\alpha\}$.

Let's prove $(\mathsf{RLD})_{\mathrm{out}}(\Omega(\mathbb{N}) \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\alpha\}) = \Omega(\mathbb{N}) \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{\alpha\}$.

Really: $(\mathsf{RLD})_{\mathrm{out}}(\Omega(\mathbb{N}) \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\alpha\}) = \bigcap \left\langle \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} \right\rangle \operatorname{up}(\Omega(\mathbb{N}) \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\alpha\}) = \bigcap \left\{ \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} (K \times \{\alpha\}) \mid K \in \operatorname{up} \Omega(\mathbb{N}) \right\}$.

$F \in \operatorname{up} \bigcap \left\{ \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} (K \times \{\alpha\}) \mid K \in \operatorname{up} \Omega(\mathbb{N}) \right\} \Leftrightarrow$
$F \in \operatorname{up} \left(\bigcap \left\{ \uparrow^{\mathbb{N}} K \mid K \in \operatorname{up} \Omega(\mathbb{N}) \right\} \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{\alpha\} \right)$ for every $F \in \mathscr{P}(\mathbb{N} \times \mathbb{N})$.
Thus
$\bigcap \left\{ \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} (K \times \{\alpha\}) \mid K \in \operatorname{up} \Omega(\mathbb{N}) \right\} =$
$\bigcap \left\{ \uparrow^{\mathbb{N}} K \mid K \in \operatorname{up} \Omega(\mathbb{N}) \right\} \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{\alpha\} = \Omega(\mathbb{N}) \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{\alpha\}$.

So $(\mathsf{RLD})_{\mathrm{out}}(\Omega(\mathbb{N}) \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\alpha\}) = \Omega(\mathbb{N}) \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{\alpha\}$.

Thus $(\mathsf{RLD})_{\mathrm{out}}(g \circ f) = \Omega(\mathbb{N}) \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{\alpha\} \neq 0^{\mathsf{RLD}(\mathbb{N};\mathbb{N})}$. □

Example 13 (FCD) does not preserve finite meets.

Proof $(\mathsf{FCD})(I^{\mathsf{RLD}(\mathbb{N})} \cap (1^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} \setminus I^{\mathsf{RLD}(\mathbb{N})})) = (\mathsf{FCD})0^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} = 0^{\mathsf{FCD}(\mathbb{N};\mathbb{N})}$.

On the other hand

$$(\mathsf{FCD})I^{\mathsf{RLD}(\mathbb{N})} \cap (\mathsf{FCD})(1^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} \setminus I^{\mathsf{RLD}(\mathbb{N})}) = I^{\mathsf{FCD}(\mathbb{N})} \cap \uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} (\mathbb{N} \times \mathbb{N} \setminus I_{\mathbb{N}}) = I^{\mathsf{FCD}}_{\Omega(\mathbb{N})} \neq 0^{\mathsf{FCD}(\mathbb{N};\mathbb{N})}$$

(used the proposition 31). □

Corollary 23 (FCD) *is not an upper adjoint (in general).*

Considering restricting polynomials (considered as reloids) to atomic filter objects, it is simple to prove that each that restriction is injective if not restricting a constant polynomial. Does this hold in general? No, see the following example:

Example 14 There exists a monovalued reloid with atomic domain which is neither injective nor constant (that is not a restriction of a constant function).

Proof (based on [16]) Consider the function $F \in \mathbb{N}^{\mathbb{N} \times \mathbb{N}}$ defined by the formula $(x; y) \mapsto x$.

Let ω_x is a non-principal atomic filter object on the vertical line $\{x\} \times \mathbb{N}$ for every $x \in \mathbb{N}$.

Let T is the collection of such sets Y that $Y \cap (\{x\} \times \mathbb{N}) \in \operatorname{up} \omega_x$ for all but finitely many vertical lines. Obviously T is a filter.

Let $\omega \in \operatorname{atoms} \operatorname{up}^{-1} T$.

For every $x \in \mathbb{N}$ we have some $Y \in T$ for which $(\{x\} \times \mathbb{N}) \cap Y = \emptyset$ and thus $(\{x\} \times \mathbb{N}) \cap \operatorname{up} \omega = \emptyset$.