Proof I will prove that quasi-complement (see [15] for the definition of quasi-complement) of the funcoid $I^{\mathsf{FCD}(\mathbb{N})}$ is not its complement. We have:

$$\begin{aligned}\left(I^{\mathsf{FCD}(\mathbb{N})}\right)^* &= \bigcup\left\{c \in \mathsf{FCD}(\mathbb{N};\mathbb{N}) \quad | \quad c \asymp I^{\mathsf{FCD}(\mathbb{N})}\right\}\\ &\supseteq \bigcup\left\{\uparrow^{\mathbb{N}}\{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}}\{\beta\} \quad | \quad \alpha,\beta \in \mathbb{N}, \uparrow^{\mathbb{N}}\{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}}\{\beta\} \asymp I^{\mathsf{FCD}(\mathbb{N})}\right\}\\ &= \bigcup\{\uparrow^{\mathbb{N}}\{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}}\{\beta\} \quad | \quad \alpha,\beta \in \mathbb{N}, \alpha \neq \beta\}\\ &= \uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} \bigcup\{\{\alpha\} \times \{\beta\} \quad | \quad \alpha,\beta \in \mathbb{N}, \alpha \neq \beta\}\\ &= \uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} (\mathbb{N} \times \mathbb{N} \setminus I_{\mathbb{N}})\end{aligned}$$

(used the corollary 10). But by proved above

$$\left(I^{\mathsf{FCD}(\mathbb{N})}\right)^* \cap I^{\mathsf{FCD}(\mathbb{N})} \neq 0^{\mathfrak{F}(\mathbb{N})}.$$

□

Example 8 There exists funcoid h such that $\operatorname{up} h$ is not a filter.

Proof Consider the funcoid $h = I^{\mathsf{FCD}}_{\Omega(\mathbb{N})}$. We have (from the proof of proposition 51) that $f \in \operatorname{up} h$ and $g \in \operatorname{up} h$, but $f \cap g = \emptyset \notin \operatorname{up} h$. □

Example 9 There exists a funcoid $h \neq 0^{\mathsf{FCD}(A;B)}$ such that $(\mathsf{RLD})_{\mathrm{out}} h = 0^{\mathsf{RLD}(A;B)}$.

Proof Consider $h = I^{\mathsf{FCD}}_{\Omega(\mathbb{N})}$. By proved above $h = f \cap g$ where $f = I^{\mathsf{FCD}(\mathbb{N})}$, $g = \uparrow^{\mathsf{FCD}(\mathbb{N};\mathbb{N})} ((\mathbb{N} \times \mathbb{N}) \setminus I_{\mathbb{N}})$.

We have $\mathrm{id}_{\mathbb{N}}, (\mathbb{N} \times \mathbb{N}) \setminus \mathrm{id}_{\mathbb{N}} \in \operatorname{up} h$.

So $(\mathsf{RLD})_{\mathrm{out}} h = \bigcap \langle \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} \rangle \operatorname{up} h \subseteq \uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})} (\mathrm{id}_{\mathbb{N}} \cap ((\mathbb{N} \times \mathbb{N}) \setminus \mathrm{id}_{\mathbb{N}})) = 0^{\mathsf{RLD}(\mathbb{N};\mathbb{N})}$; and thus $(\mathsf{RLD})_{\mathrm{out}} h = 0^{\mathsf{RLD}(\mathbb{N};\mathbb{N})}$. □

Example 10 There exists a funcoid h such that $(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{out}} h \neq h$.

Proof It follows from the previous example. □

Example 11 $(\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD}) f \neq f$ for some convex reloid f.

Proof Let $f = I^{\mathsf{RLD}(\mathbb{N})}$. Then $(\mathsf{FCD}) f = I^{\mathsf{FCD}(\mathbb{N})}$. Let a be some non-trivial atomic f.o. Then $(\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD}) f \supseteq a \times^{\mathsf{RLD}} a \nsubseteq I^{\mathsf{RLD}(\mathbb{N})}$ and thus $(\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD}) f \nsubseteq f$. □

Remark 10 Before I found the last counter-example, I thought that $(\mathsf{RLD})_{\mathrm{in}}$ is an isomorphism from the set of of funcoids to the set of convex reloids. As this conjecture failed, we need an other way to characterize the set of reloids isomorphic to funcoids.