Example 1 There exist a funcoid f and a set S of funcoids such that $f \cap \bigcup S \neq \bigcup \langle f\cap \rangle S$.

Proof Let $f = \Delta \times^{\mathsf{FCD}} \uparrow^{\mathfrak{F}(\mathbb{R})} \{0\}$ and $S = \left\{ \uparrow^{\mathsf{FCD}(\mathbb{R};\mathbb{R})} ((\varepsilon; +\infty) \times \{0\}) \quad | \quad \varepsilon > 0 \right\}$. Then $f \cap \bigcup S = (\Delta \times^{\mathsf{FCD}} \uparrow^{\mathfrak{F}(\mathbb{R})} \{0\}) \cap \uparrow^{\mathsf{FCD}(\mathbb{R};\mathbb{R})} ((0; +\infty) \times \{0\}) = (\Delta \cap \uparrow^{\mathfrak{F}(\mathbb{R})} (0; +\infty)) \times^{\mathsf{FCD}} \uparrow^{\mathfrak{F}(\mathbb{R})} \{0\} \neq 0^{\mathsf{FCD}(\mathbb{R};\mathbb{R})}$ while $\bigcup \langle f\cap \rangle S = \bigcup \left\{ 0^{\mathsf{FCD}(\mathbb{R};\mathbb{R})} \right\} = 0^{\mathsf{FCD}(\mathbb{R};\mathbb{R})}$. □

Example 2 There exist a set R of funcoids and a funcoid f such that $f \circ \bigcup R \neq \bigcup \langle f\circ \rangle R$.

Proof Let $f = \Delta \times^{\mathsf{FCD}} \{0\}$, $R = \left\{ \{0\} \times^{\mathsf{FCD}} (\varepsilon; +\infty) \quad | \quad \varepsilon \in \mathbb{R} \right\}$.

We have $\bigcup R = \{0\} \times^{\mathsf{FCD}} (0; +\infty)$; $f \circ \bigcup R = \uparrow^{\mathsf{FCD}(\mathbb{R};\mathbb{R})} (\{0\} \times \{0\}) \neq 0^{\mathsf{FCD}(\mathbb{R};\mathbb{R})}$ and $\bigcup \langle f\circ \rangle R = \bigcup \left\{ 0^{\mathsf{FCD}(\mathbb{R};\mathbb{R})} \right\} = 0^{\mathsf{FCD}(\mathbb{R};\mathbb{R})}$. □

Example 3 There exist a set R of funcoids and f.o. $\mathcal{X}$ and $\mathcal{Y}$ such that

1. $\mathcal{X} \left[\bigcup R \right] \mathcal{Y} \wedge \nexists f \in R : \mathcal{X} [f] \mathcal{Y}$;
2. $\langle \bigcup R \rangle \mathcal{X} \supset \bigcup \{ \langle f \rangle \mathcal{X} \quad | \quad f \in R \}$.

Proof

1. Let $\mathcal{X} = \Delta$ and $\mathcal{Y} = 1^{\mathfrak{F}(\mathbb{R})}$. Let $R = \left\{ \uparrow^{\mathsf{FCD}(\mathbb{R};\mathbb{R})} ((\varepsilon; +\infty) \times \mathbb{R}) \quad | \quad \varepsilon \in \mathbb{R}, \varepsilon > 0 \right\}$. Then $\bigcup R = \uparrow^{\mathsf{FCD}(\mathbb{R};\mathbb{R})} ((0; +\infty) \times \mathbb{R})$. So $\mathcal{X} \left[\bigcup R \right] \mathcal{Y}$ and $\forall f \in R : \neg (\mathcal{X} [f] \mathcal{Y})$.
2. With the same $\mathcal{X}$ and R we have $\langle \bigcup R \rangle \mathcal{X} = \mathbb{R}$ and $\langle f \rangle \mathcal{X} = 0^{\mathfrak{F}(\mathbb{R})}$ for every $f \in R$, thus $\bigcup \{ \langle f \rangle \mathcal{X} \quad | \quad f \in R \} = 0^{\mathfrak{F}(\mathbb{R})}$.

□

Theorem 78 *For a f.o. a we have $a \times^{\mathsf{RLD}} a \subseteq I^{\mathsf{RLD}(\mathrm{Base}(a))}$ only in the case if $a = 0^{\mathfrak{F}(\mathrm{Base}(a))}$ or a is a trivial atomic f.o. (that is corresponds to an one-element set).*

Proof If $a \times^{\mathsf{RLD}} a \subseteq I^{\mathsf{RLD}(\mathrm{Base}(a))}$ then exists $m \in \mathrm{up}(a \times^{\mathsf{RLD}} a)$ such that $m \subseteq I_{\mathrm{Base}(a)}$. Consequently exist $A, B \in \mathrm{up}\, a$ such that $A \times B \subseteq I_{\mathrm{Base}(a)}$ what is possible only in the case when $\uparrow^{\mathrm{Base}(a)} A = \uparrow^{\mathrm{Base}(a)} B = a$ and $A = B$ is an one-element set or empty set. □

Corollary 22 *Reloidal product of a non-trivial atomic filter object with itself is non-atomic.*

Proof Obviously $(a \times^{\mathsf{RLD}} a) \cap I^{\mathsf{RLD}(\mathrm{Base}(a))} \neq 0^{\mathfrak{F}(\mathrm{Base}(a))}$ and $(a \times^{\mathsf{RLD}} a) \cap I^{\mathsf{RLD}(\mathrm{Base}(a))} \subset a \times^{\mathsf{RLD}} a$. □