Proposition 49 *Let A be a set. The f.o. $\uparrow^{\operatorname{Ob}\mu} A$ is connected regarding an endo-funcoid μ iff*

$$\forall X, Y \in \mathscr{P}(\operatorname{Ob}\mu) \setminus \{\emptyset\} : (X \cup Y = A \Rightarrow X \, [\mu]^* \, Y).$$

Proof

$\Rightarrow$ Obvious.

$\Leftarrow$ It follows from co-separability of filter objects.

□

Theorem 75 *The following are equivalent for every set A and binary relation μ on a set U:*

1. *A is connected regarding binary relation μ.*
2. *$\uparrow^U A$ is connected regarding $\uparrow^{\mathsf{RLD}(U;U)} \mu$.*
3. *$\uparrow^U A$ is connected regarding $\uparrow^{\mathsf{FCD}(U;U)} \mu$.*

Proof

(1)⇔(2) $S^*\left(\uparrow^{\mathsf{RLD}(U;U)} \mu \cap \left(\uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A\right)\right) = S^*\left(\uparrow^{\mathsf{RLD}(U;U)} (\mu \cap (A \times A))\right) = \uparrow^{\mathsf{RLD}(U;U)} S(\mu \cap (A \times A))$. So $S^*\left(\uparrow^{\mathsf{RLD}(U;U)} \mu \cap (\uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A)\right) \supseteq \uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A \Leftrightarrow \uparrow^{\mathsf{RLD}(U;U)} S(\mu \cap (A \times A)) \supseteq \uparrow^{\mathsf{RLD}(U;U)} (A \times A) = \uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A$.

(1)⇔(3) It follows from the previous proposition.

□

Next is conjectured a statement more strong than the above theorem:

Conjecture 21 *Let $\mathcal{A}$ is a f.o. on a set U and F is a binary relation on U.*

$\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{FCD}(U;U)} F$ iff $\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{RLD}(U;U)} F$.

Obvious 31. A filter object $\mathcal{A}$ is connected regarding a reloid μ iff it is connected regarding the reloid $\mu \cap (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A})$.

Obvious 32. A filter object $\mathcal{A}$ is connected regarding a funcoid μ iff it is connected regarding the funcoid $\mu \cap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{A})$.

Theorem 76 *A filter object $\mathcal{A}$ is connected regarding a reloid f iff $\mathcal{A}$ is connected regarding every $F \in \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Ob} f;\operatorname{Ob} f)} \right\rangle \operatorname{up} f$.*

Proof