$$
\begin{aligned}
S(\mu)\circ S(\mu) &= \mu^0\circ S(\mu)\cup\mu\circ S(\mu)\cup\mu^2\circ S(\mu)\cup\ldots\\
&= (\mu^0\cup\mu^1\cup\mu^2\cup\ldots)\cup(\mu^1\cup\mu^2\cup\mu^3\cup\ldots)\cup(\mu^2\cup\mu^3\cup\mu^4\cup\ldots)\\
&= \mu^0\cup\mu^1\cup\mu^2\cup\ldots\\
&= S(\mu).
\end{aligned}
$$

□

7.3 Connectedness regarding binary relations

Before going to research connectedness for funcoids and reloids we will excurse into the basic special case of connectedness regarding binary relations on a set $\mho$.

Definition 58 *A set A is called **(strongly) connected** regarding a binary relation μ when*

$$\forall X\in\mathscr{P}(\operatorname{dom}\mu)\setminus\{\emptyset\}, Y\in\mathscr{P}(\operatorname{im}\mu)\setminus\{\emptyset\} : (X\cup Y = A\Rightarrow X\ [\mu]\ Y).$$

Let $\mho$ be a set.

Definition 59 ***Path*** *between two elements $a,b\in\mho$ in a set $A\subseteq\mho$ through binary relation μ is the finite sequence $x_0\ldots x_n$ where $x_0=a$, $x_n=b$ for $n\in\mathbb{N}$ and $x_i(\mu\cap A\times A)x_{i+1}$ for every $i=0,\ldots,n-1$. n is called path **length**.*

Proposition 44 *There exists path between every element $a\in\mho$ and that element itself.*

Proof It is the path consisting of one vertex (of length 0). □

Proposition 45 *There is a path from element a to element b in a set A through a binary relation μ iff $a\ (S(\mu\cap A\times A))\ b$ (that is $(a,b)\in S(\mu\cap A\times A)$).*

Proof

⇒ If a path from a to b exists, then $\{b\}\subseteq\langle(\mu\cap A\times A)^n\rangle\{a\}$ where n is the path length. Consequently $\{b\}\subseteq\langle S(\mu\cap A\times A)\rangle\{a\}$; $a\ (S(\mu\cap A\times A))\ b$.

⇐ If $a\ (S(\mu\cap A\times A))\ b$ then exists $n\in\mathbb{N}$ such that $a\ (\mu\cap A\times A)^n\ b$. By definition of composition of binary relations this means that there exist finite sequence $x_0\ldots x_n$ where $x_0=a$, $x_n=b$ for $n\in\mathbb{N}$ and $x_i\ (\mu\cap A\times A)\ x_{i+1}$ for every $i=0,\ldots,n-1$. That is there is path from a to b.

□