7.1 Some lemmas

Lemma 6 *If* $\neg(A\ [f]^*\ B) \wedge A \cup B \in \operatorname{up}(\operatorname{dom} f \cup \operatorname{im} f)$ *then* f *is closed on* $\uparrow^U A$ *for a funcoid* $f \in \mathsf{FCD}(U; U)$ *and sets* $A, B \in \mathscr{P}U$ *(for every small set* U*).*

Proof Let $A \cup B \in \operatorname{up}(\operatorname{dom} f \cup \operatorname{im} f)$. $\neg(A\ [f]^*\ B) \Leftrightarrow \uparrow^U B \cap \langle f \rangle \uparrow^U A = 0^{\mathfrak{F}(U)} \Rightarrow (\operatorname{dom} f \cup \operatorname{im} f) \cap \uparrow^U B \cap \langle f \rangle^* A = 0^{\mathfrak{F}(U)} \Rightarrow ((\operatorname{dom} f \cup \operatorname{im} f) \setminus \uparrow^U A) \cap \langle f \rangle^* A = 0^{\mathfrak{F}(U)} \Leftrightarrow \langle f \rangle^* A \subseteq \uparrow^U A$. □

Corollary 19 *If* $\neg(A\ [f]^*\ B) \wedge A \cup B \in \operatorname{up}(\operatorname{dom} f \cup \operatorname{im} f)$ *then* f *is closed on* $\uparrow^U (A \setminus B)$ *for a funcoid* f *and sets* $A, B \in \mathscr{P}U$ *(for every small set* U*).*

Proof Let $\neg(A\ [f]^*\ B) \wedge A \cup B \in \operatorname{up}(\operatorname{dom} f \cup \operatorname{im} f)$. Then $\neg((A \setminus B)\ [f]^*\ B) \wedge \uparrow^U ((A \setminus B) \cup B) \in \operatorname{up}(\operatorname{dom} f \cup \operatorname{im} f)$. □

Lemma 7 *If* $\neg(A\ [f]^*\ B) \wedge A \cup B \in \operatorname{up}(\operatorname{dom} f \cup \operatorname{im} f)$ *then* $\neg(A\ [f^n]^*\ B)$ *for every whole positive* n*.*

Proof Let $\neg(A\ [f]^*\ B) \wedge A \cup B \in \operatorname{up}(\operatorname{dom} f \cup \operatorname{im} f)$. From the above lemma $\langle f \rangle^* A \subseteq \uparrow^U A$. $\uparrow^U B \cap \langle f \rangle \uparrow^U A = 0^{\mathfrak{F}(U)}$, consequently $\langle f \rangle^* A \subseteq \uparrow^U (A \setminus B)$. Because (by the above corollary) f is closed on $\uparrow^U (A \setminus B)$, then $\langle f \rangle \langle f \rangle \uparrow^U A \subseteq \uparrow^U (A \setminus B)$, $\langle f \rangle \langle f \rangle \langle f \rangle \uparrow^U A \subseteq \uparrow^U (A \setminus B)$, etc. So $\langle f^n \rangle \uparrow^U A \subseteq \uparrow^U (A \setminus B)$, $\uparrow^U B \asymp \langle f^n \rangle \uparrow^U A$, $\neg(A\ [f^n]^*\ B)$. □

7.2 Endomorphism series

Definition 56 $S_1(\mu) \stackrel{\text{def}}{=} \mu \cup \mu^2 \cup \mu^3 \cup \ldots$ *for an endomorphism* μ *of a precategory with countable join of morphisms.*

Definition 57 $S(\mu) \stackrel{\text{def}}{=} \mu^0 \cup S_1(\mu) = \mu^0 \cup \mu \cup \mu^2 \cup \mu^3 \cup \ldots$ *where* $\mu^0 \stackrel{\text{def}}{=} I_{\operatorname{Ob} \mu}$ *(identity morphism for the object* $\operatorname{Ob} \mu$*) where* $\operatorname{Ob} \mu$ *is the object of endomorphism* μ *for an endomorphism* μ *of a category with countable join of morphisms.*

I call S_1 and S **endomorphism series**.

We will consider the collection of all binary relations (on a set $\mho$), as well as the collection of all funcoids and the collection of all reloids on a fixed set, as categories with single object $\mho$ and the identity morphisms $I_\mho$, $I^{\mathsf{FCD}(\mho)}$, $I^{\mathsf{RLD}(\mho)}$.

Proposition 43 *The relation* $S(\mu)$ *is transitive for the category of binary relations.*

Proof