6.1.2 Proximity spaces

Let μ and ν are proximity (nearness) spaces (which I consider a special case of funcoids). By definition a function f is a proximity-continuous map (also called equivicontinuous) from μ to ν iff

$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : (X\ [\mu]^*\ Y \Rightarrow \langle f\rangle X\ [\nu]^*\ \langle f\rangle Y).$$

Equivalently transforming this formula we get (writing $\uparrow$ instead of $\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)}$ for brevity):

$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : (X\ [\mu]^*\ Y \Rightarrow \langle f\rangle Y \cap \langle\nu\rangle\langle f\rangle X \neq 0^{\mathfrak{F}(\operatorname{Dst}\nu)});$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : (X\ [\mu]^*\ Y \Rightarrow \langle f\rangle Y \cap \langle\nu\circ\uparrow f\rangle^* X \neq 0^{\mathfrak{F}(\operatorname{Dst}\nu)});$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : (X\ [\mu]^*\ Y \Rightarrow X\ [\nu\circ\uparrow f]^*\ \langle f\rangle Y);$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : (X\ [\mu]^*\ Y \Rightarrow \langle f\rangle Y\ \left[(\nu\circ\uparrow f)^{-1}\right]^*\ X);$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : (X\ [\mu]^*\ Y \Rightarrow \langle f\rangle Y\ \left[(\uparrow f)^{-1}\circ\nu^{-1}\right]^*\ X);$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : \left(X\ [\mu]^*\ Y \Rightarrow \uparrow^{\mathfrak{F}(\operatorname{Src}\mu)} X \cap \left\langle(\uparrow f)^{-1}\circ\nu^{-1}\right\rangle\langle f\rangle Y \neq 0^{\mathfrak{F}(\operatorname{Src}\mu)}\right);$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : \left(X\ [\mu]^*\ Y \Rightarrow \uparrow^{\mathfrak{F}(\operatorname{Src}\mu)} X \cap \left\langle(\uparrow f)^{-1}\circ\nu^{-1}\circ\uparrow f\right\rangle^* Y \neq 0^{\mathfrak{F}(\operatorname{Src}\mu)}\right);$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : \left(X\ [\mu]^*\ Y \Rightarrow Y\ \left[(\uparrow f)^{-1}\circ\nu^{-1}\circ\uparrow f\right]^*\ X\right);$$
$$\forall X \in \mathscr{P}(\operatorname{Src}\mu), Y \in \mathscr{P}(\operatorname{Dst}\mu) : \left(X\ [\mu]^*\ Y \Rightarrow X\ \left[(\uparrow f)^{-1}\circ\nu\circ\uparrow f\right]^*\ Y\right);$$
$$\mu \subseteq (\uparrow f)^{-1}\circ\nu\circ\uparrow f.$$

So a function f is proximity-continuous iff $\mu \subseteq \left(\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)} f\right)^{-1}\circ\nu\circ\uparrow^{\mathsf{FCD}(\operatorname{Src}\mu;\operatorname{Dst}\nu)} f$.

6.1.3 Uniform spaces

Uniform spaces are a special case of reloids.

Let μ and ν are uniform spaces. By definition a function f is a uniformly continuous map from μ to ν iff

$$\forall\epsilon \in \operatorname{up}\nu\exists\delta \in \operatorname{up}\mu\forall(x;y) \in \delta : (fx;fy) \in \epsilon.$$

Equivalently transforming this formula we get:

$$\forall\epsilon \in \operatorname{up}\nu\exists\delta \in \operatorname{up}\mu\forall(x;y) \in \delta : \{(fx;fy)\} \subseteq \epsilon;$$
$$\forall\epsilon \in \operatorname{up}\nu\exists\delta \in \operatorname{up}\mu\forall(x;y) \in \delta : f\circ\{(x;y)\}\circ f^{-1} \subseteq \epsilon;$$
$$\forall\epsilon \in \operatorname{up}\nu\exists\delta \in \operatorname{up}\mu : f\circ\delta\circ f^{-1} \subseteq \epsilon;$$
$$\forall\epsilon \in \operatorname{up}\nu : \uparrow^{\mathsf{RLD}(\operatorname{Dst}\mu;\operatorname{Dst}\nu)} f\circ\mu\circ\left(\uparrow^{\mathsf{RLD}(\operatorname{Dst}\mu;\operatorname{Dst}\nu)} f\right)^{-1} \subseteq \uparrow^{\mathsf{RLD}(\operatorname{Src}\nu;\operatorname{Dst}\nu)} \epsilon;$$
$$\uparrow^{\mathsf{RLD}(\operatorname{Dst}\mu;\operatorname{Dst}\nu)} f\circ\mu\circ\left(\uparrow^{\mathsf{RLD}(\operatorname{Dst}\mu;\operatorname{Dst}\nu)} f\right)^{-1} \subseteq \nu.$$

So a function f is uniformly continuous iff $\uparrow^{\mathsf{RLD}(\operatorname{Dst}\mu;\operatorname{Dst}\nu)} f\circ\mu\circ\left(\uparrow^{\mathsf{RLD}(\operatorname{Dst}\mu;\operatorname{Dst}\nu)} f\right)^{-1} \subseteq \nu$.